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Surface mesh denoising with normal tensor framework

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ABSTRACT

In this paper, we propose a novel method for feature-preserving mesh denoising based on the normal tensor framework. We utilize the normal tensor voting directly for the mesh denoising whose eigenvalues and eigenvectors are used for detecting saliency, and introduce an algorithm that updates a vertex by the Laplacian of curvature which minimizes a difference of the curvature in one neighborhood. By connecting the feature saliency with a distance metric in the normal tensor space, our algorithm preserves sharp features more robustly and clearly for noisy mesh data. Comparing our method with the existing ones, we demonstrate the effectiveness of our algorithm against some synthetic noisy data and real-world scanned data.

1. Introduction

Surface mesh denoising has been an important research area and its goal is to eliminate noise or spurious information on the mesh while preserving original features. Therefore, a good denoising algorithm should be able to remove the noise while retaining original features as clearly as possible. In particular, to preserve sharp features such as crease edges and corners indicated in Fig. 1, is our great concern, since they are often blurred if no special care is taken.

Various approaches have still been proposed to tackle this problems on end, and are classified into three categories, depending on whether the process is performed under a deterministic, image filtering [7,17,34] or probabilistic point of view [6,23]. In this paper, we focus on a deterministic framework which is governed by an anisotropic diffusion equation. Most of their approaches use a weighed Laplacian smoothing. Representative works are Taubin's signal processing technique called by $\lambda|\mu$ method [30] and Desbrun's *mean curvature flow* method [5]. But the Laplacian-based methods have two main drawbacks:

(i) smoothing side-effects which blur sharp features,
(ii) shrinkage of the shape as a whole.

In order to avoid (i), anisotropic diffusion is discussed by many researchers [1,3,4,8,35]. The basic mechanism is the directional smoothing that at the smaller principal curvature value errors are reduced and at the larger value less. With respect to (ii), Kobbelt et al. [12] applied a surface fairing in CAGD [9], and proposed a discrete fairing method by the second order Laplacian in order to prevent from shrinking in denoising process. But their approach is not enough for mesh denoising with sharp features. In addition, we add the following problem to the above two:

(iii) cumbersome setting of tuning parameters.

For example, in the algorithms using a Gaussian weighting function, the band width becomes an important parameter for getting better results, but the way how to decide the value is hardly discussed in detail and treated as a thing that should be properly set by users suited for the level of noise.

In this paper, we propose a novel method for the feature-preserving mesh denoising based on the normal tensor which is constructed by a sum of covariance matrices of unit facet normal vectors in one-ring neighborhood. The main idea behind our approach is followed by two things. At first, we recognize the characteristic features

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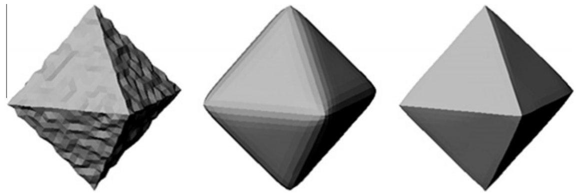

Fig. 1. Smoothing and denoising a noisy octahedron (|V| = 1026,|F| = 2048). From left to right, Noisy data, laplacian smoothing and our denoising.

via surface normal not via curvature or higher order differential invariants [14], since they are sensitive to noise and cannot be directly computed on noisy data. Furthermore, sharp features cannot be primarily expressed by a differentiable manifold, so we cannot represent them precisely as a smooth surface. As reported in [2,28], the surface normal describes the feature saliencies well though it is lower order derivative than curvature. Secondly, in order to prevent from the shrinking problem, we also use a surface fairing technique as Kobbelt et al. [12], and apply it to the feature-preserving mesh denoising, though the existing methods does not often use it. In general, different from Laplacian based smoothing, fairing is related to aesthetics and is often processed with higher order derivative such as the second order Laplacian. Therefore, by the same reason as stated above, we develop a robust algorithm for noise to realize the fairing technique.

Our contributions are as follows: by connecting the feature saliency based on the normal tensor framework, which is proposed by Medioni et al. [19] and Page et al. [24], with a distance metric in the normal tensor space, we have realized the following items:

- a novel algorithm that updates a vertex by the Laplacian of curvature which minimizes a difference of the curvature in one neighborhood, preserves sharp features more robustly and clearly for noisy mesh data,
- the proposed method prevents from the shrinkage problem due to the property of Laplacian of curvature,
- our weighting function is simply designed, and the denoising algorithm contains only one tuning parameter that defines a crease angle.

The rest of the paper is organized as follows: Section 2 presents a brief overview of related works. The working principle of normal tensor is explained in Section 3. In Section 4, we introduce our method which simultaneously smoothes the geometric flow and noisy mesh. Results and some comparison with other schemes are done in Section 5 and finally we conclude the paper in Section 6.

Before proceeding to Section 2, we define some notations used in this paper.

Notations: We treat a triangle mesh denoted by $M(V, F)$, where V and F indicate mesh vertices $V = \{V_i | i = 1, 2, \ldots, |V|\}$ and triangle facets $F = \{F_i | i = 1, 2, \ldots, |F|\}$, respectively. Here $|\cdot|$ denotes the cardinality of a set. Vertices and facets have each unit normal vector $\boldsymbol{N}^v$ and $\boldsymbol{N}^f$ respectively. And 1-ring neighborhood of a vertex V_i has two cases; one is vertex neighbor indices denoted by $\Lambda_V(i)$ and the other is facet indices $\Lambda_F(i)$. ∂F denotes a set of edges that constitute a facet boundary. An illustration is shown in Fig. 2.

2. Related work

The governing equation in mesh denoising where we focus on as a deterministic framework, is described by the following diffusion equation:

$$\begin{cases} \partial u(x,t) = div(D(x)\nabla u(x,t)) \\ u(x,0) = f(x), \end{cases} \tag{1}$$

where $D(x)$ is diffusion coefficient, $u(x, t)$ is a certain function such as concentration of a matter and $x \in R^d$, $t > 0$. We show some mathematical foundations for the functions related to our proposed approach below.

2.1. Isotropic diffusion

Here we consider $d = 1$, and $D(x)$ is a constant D_0. Eq. (1) is reduced to $\partial_t u(x, t) = D_0 \partial_{xx} u(x, t)$, which results in the following equation with a simple finite difference scheme:

$$u(x, t+\Delta t) = u(x,t) + \frac{D_0 \delta t}{(\Delta x)^2} \sum_{\tilde{x} \in \Lambda_x} (u(\tilde{x}, t) - u(x,t)),$$

where $\Lambda_x \equiv \{x + \Delta x, x - \Delta x\}$.

We apply the above finite difference equation to an unstructured mesh, that is, replace Λ_x and $u(x,t)$ with $\Lambda_V(i)$ and mesh vertex V_i respectively. Thus we obtain the following vertex update equation:

$$V_i^{n+1} \leftarrow V_i^n + \frac{\lambda}{\sum\limits_{j \in \Lambda_V(i)} w_{ij}} \sum_{j \in \Lambda_V(i)} w_{ij} (V_j^n - V_i^n), \tag{2}$$

where $\lambda > 0$ is the iteration step size, which should be a small value for stable calculation. Eq. (2) is well known as Laplacian smoothing, and the weight function w_{ij} plays an important role, since simple functions such as angular- or area-weight have lost and blurred the feature characteristics due to a diffusion process. On the other hand, the counterpart of the Euclidian Laplacian Δ on a smooth surface is the *Laplace–Beltrami operator* Δ_M. Thus we obtain a geometric diffusion equation [27]

$$\partial_t x = \Delta_M x, \tag{3}$$

where x is a point on the manifold M. From differential geometry [33], we know that the mean curvature vector $K^H \boldsymbol{n}$ equals the *Laplace–Beltrami operator* on a surface manifold:

$$-K^H \boldsymbol{n} = \Delta_M x.$$

Thus the geometric diffusion Eq. (3) is equivalent to the *mean curvature flow* (MCF) given by

$$\partial_t x = -K^H \boldsymbol{n}.$$

In mesh area, Desbrun et al. [5] proposed the MCF smoothing method using a discrete mean curvature vector $K_i^H \boldsymbol{n}_i$ given by cotangent formula: