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Point-augmented biquadratic C^1 subdivision surfaces ☆

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ABSTRACT

Shape artifacts, especially for convex input polyhedra, make Doo and Sabin's generalization of bi-quadratic (bi-2) subdivision surfaces unattractive for general design. Rather than tuning the eigenstructure of the subdivision matrix, we improve shape by adding a point and enriching the refinement rules. Adding a guiding point can also yield a polar bi-2 subdivision algorithm. Both the augmented and the polar bi-2 subdivision are complemented by a new Primal Bi-2 Subdivision scheme. All surfaces are C^1 and can be combined.

1. Introduction

Primal Catmull-Clark subdivision (CC [2]) and dual Doo-Sabin subdivision (DS [3]) surfaces were introduced back-to-back in 1978 in order to extend smooth surfaces beyond tensor-product control nets. CC subdivision allows extraordinary control points where $n \neq 4$ quadrilaterals meet. DS subdivision allows extraordinary facets with $n \neq 4$ vertices.

CC subdivision has been widely popular with artists in the entertainment industry. Interpretation of the vertices of a regular CC mesh as control points of bi-cubic (bi-3) tensor-product B-splines yields a surface with smoothly-varying normals almost everywhere. By contrast, DS subdivision has not seen main-stream applications. This can be blamed on the underlying bi-2 tensor-product B-splines whose parameter lines join less gracefully: the normals vary only continuously between polynomial pieces and the images of parameter-lines are piecewise parabolas so that inflections along parameter lines can only occur at joints. However, more damaging are the under-documented shape defects near the facet-center limit points of DS subdivision Fig. 1(b) and (d).

The goal of this paper is to revisit and rehabilitate Doo-Sabin subdivision near these extraordinary points. Sophisticated tuning strategies have been developed in the literature [11,10,1] to improve the eigenspectrum near extraordinary point s. By contrast, we aim to improve Doo-Sabin subdivision by adding a point and enriching the refinement rules. Our interest was triggered by recent high-quality multi-sided spline constructions based on guide surfaces (e.g. [7]). This approach uses the characteristic map and the extraordinary point of CC subdivision. Bi-2 subdivision would provide a simpler instance, on which to base a G^1 construction. Unfortunately, the strong artifacts of Doo-Sabin subdivision mean that the subdivision itself has to be improved before its components can be used.

Our second goal was to extend bi-2 subdivision to operate naturally on quad meshes; and to adapt it to a polar

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layout suitable for high valences. That is, the contributions of this submission are:

- Introducing a new point-augmented subdivision approach to remove artifacts present in Doo-Sabin subdivision (Fig. 2(a)).
- Adapting bi-2 subdivision to be primal, quad-based (Fig. 2(b)).
- Introducing bi-2 subdivision for polar configurations (Fig. 2(c)).

1.1. Overview

Section 2 motivates, describes and analyzes the new Point-Augmented Bi-2 Subdivision devised to improve Doo-Sabin subdivision. Section 3 adds Polar Bi-2 Subdivision and Section 4 presents Primal Bi-2 Subdivision, a variant suitable for operating directly on quad meshes.

2. Point-Augmented Bi-2 Subdivision

The shape artifacts of Doo-Sabin subdivision (DS) include, for convex input, flatness at the extraordinary point and oscillation nearby (see Fig. 1). These and many other examples suggest that information is missing near the facet center. Our remedy is to add a central point $\mathbf{p}$ to the original control-net, see e.g. Fig. 3(b). Such an additional point could be treated as a user-accessible shape parameter. However, $\mathbf{p}$ is extremely difficult to set by hand without damaging the surface quality.

Both for derivation and analysis, we view subdivision as a recipe for generating a sequence of nested surface rings (see Fig. 2). In Fig. 4, black disks denote the standard control-net of Doo-Sabin subdivision, the *DS-net*. Labels per sector, the point $\mathbf{p}$ and one (green) ring of regular bi-2 patches are displayed. Superscripts will indicate the subdivision level; but wherever it is unabiguous, we omit the superscript.

Structural symmetry and simplicity imply that $\mathbf{p}$ be a linear combination of the averages of the control points closest to the center:

$$\mathbf{p} := \mathbf{p}^0 := \mu_n \bar{\mathbf{a}}^0 + (1-\mu)\frac{\bar{\mathbf{b}}^0 + \bar{\mathbf{d}}^0}{2}, \quad \mu_n := \begin{cases} 3/5, & n=3, \\ 2\frac{n-2}{n}, & n \geqslant 4. \end{cases} \tag{1}$$

$$\bar{\mathbf{a}}^k := \sum_{i=0}^{n-1} \mathbf{a}_i^k/n, \quad \bar{\mathbf{b}}^k := \sum_{i=0}^{n-1} \mathbf{b}_i^k/n, \quad \bar{\mathbf{d}}^k := \sum_{i=0}^{n-1} \mathbf{d}_i^k/n. \tag{2}$$

The key lies in setting μ_n. We determined μ_n to be simple while matching values that optimize reflection lines over a large set of challenging input nets. As intended, this choice extrapolates convex DS-nets when $n > 4$.

We derive the **refinement rules** of the new Point-Augmented Bi-2 Subdivision (PGS) by splitting the regular bi-2 patches into 2×2 subpatches in Bernstein-Bézier form (BB-form). Their inner BB-coefficients are the points of the refined control net, e.g. those marked as black disks with labels $5, \ldots, 9$ in Fig. 4(a). New points $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, labelled $2, 3, 4$ in Fig. 4(a) are obtained as inner BB-coefficients of bi-2 patches that C^1-extend the existing bi-2 complex towards the center, i.e. the stencils are those of standard binary refinement. With α, w_1, w_2 still to be determined by (8)–(10), the added point, its n direct neighbors $\mathbf{a}$ (labelled 1 in Fig. 4(a)) and its indirect neighbors are set to

$$\mathbf{p}^{new} := (1-\alpha)\mathbf{p} + \alpha\bar{\mathbf{a}}, \tag{3}$$

$$\mathbf{a}_i^{new} := (1 - w_1 - 2w_2)\mathbf{p} + w_1\mathbf{a}_i + w_2(\mathbf{a}_{i-1} + \mathbf{a}_{i+1}). \tag{4}$$

$$\mathbf{b}_i^{new} := (9\mathbf{a}_i + 3\mathbf{a}_{i-1} + 3\mathbf{b}_i + \mathbf{d}_{i-1})/16,$$

$$\mathbf{d}_i^{new} := (9\mathbf{a}_i + 3\mathbf{a}_{i+1} + 3\mathbf{d}_i + \mathbf{b}_{i+1})/16.$$

Standard analysis of subdivision schemes following [9] applies the Discrete Fourier Transform to the $10n \times 10n$ circulant subdivision matrix A (where $\mathbf{p}/n$ is replicated n times). With the k-th Fourier block denoted by $\hat{A}_k$, the characteristic polynomials $D_k(\lambda)$ of $\hat{A}_k$ are, up to constant scaling, with $d(\lambda) := (\lambda - \frac{1}{4})(\lambda - \frac{1}{8})(\lambda - \frac{1}{16})\lambda^5$,

$$D_0(\lambda) = (\lambda - 1)d(\lambda)(\lambda - w_1 - 2w_2 + \alpha); \tag{5}$$

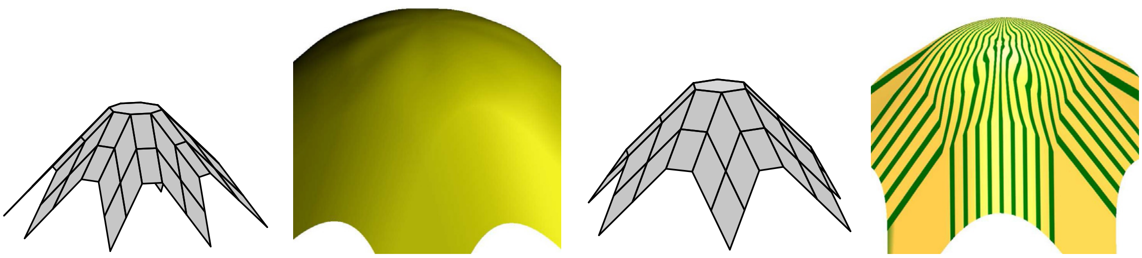
$$\text{for } k = 1, \ldots, n-1 \quad \mathrm{c}_k := \cos(2\pi k/n),$$

$$D_k(\lambda) = d(\lambda)\lambda(\lambda - w_1 - 2w_2\mathrm{c}_k). \tag{6}$$

The eigenvalue $\lambda_{sub} := w_1 + 2w_2\mathrm{c}_1$ of the blocks $\hat{A}_1$ and $\hat{A}_{n-1}$ is subdominant for suitable choice of w_1, w_2 and the corresponding characteristic ring (map) is injective (as will be clear by substituting λ_{sub} from (10) for λ in the expression [9, 6.21, p118]). The limit of the central point is computed from the left eigenvector of the eigenvalue 1 (see e.g. [4]):

$$eop := (1-\tau)\mathbf{p} + \tau\bar{\mathbf{a}}_i, \quad \tau := \frac{\alpha}{1 + \alpha - w_1 - 2w_2}. \tag{7}$$

We now set the parameters α, w_1, w_2. As shown in the Appendix, the rules for mapping a DS-net to a new DS-net in their current general form are non-stationary.



(a) Convex, $n = 8$ (b) Oscillation near and flatness at central point (c) Convex, $n = 6$ (d) Highlight lines

Fig. 1. Doo-Sabin subdivision artifacts: (b) plain rendering shows artifacts for $n = 8$ and (d) highlight shading reveals them for $n = 6$.