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Structure-aligned guidance estimation in surface parameterization using eigenfunction-based cross field ☆

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ABSTRACT

In this paper, we present a structure-aligned approach for surface parameterization using eigenfunctions from the Laplace–Beltrami operator. Several methods are designed to combine multiple eigenfunctions using isocontours or characteristic values of the eigenfunctions. The combined gradient information of eigenfunctions is then used as a guidance for the cross field construction. Finally, a global parameterization is computed on the surface, with an anisotropy enabled by adapting the cross field to non-uniform parametric line spacings. By combining the gradient information from different eigenfunctions, the generated parametric lines are automatically aligned with the structural features at various scales, and they are insensitive to local detailed features on the surface when low-mode eigenfunctions are used.

1. Introduction

Surface parameterization is of great importance for many applications, such as quadrilateral meshing [1], texture mapping and synthesis [2,3]. An important issue for surface parameterization is how to align parametric lines with the feature directions. Some simplification techniques [4–6] were developed to generate very coarse domain meshes with a good user control. Although feature alignment was achieved in a certain degree [4], it is difficult to control the simplification process to preserve surface features. Using the harmonic field [7,8], features can be captured, but feature alignment is limited due to the difficulty in generating the field and placing singularities. In recent years, methods based on the cross field have been introduced [7,9–12]. Generally, the captured features in the cross field are represented by the principal curvatures, which are sensitive to the local detailed features and may fail in capturing structural features of an object at desired scales.

Eigenfunctions of the Laplace–Beltrami operator (LBO) are well-known for their property of capturing the shape behavior and structural feature of an object [13–16]. Eigenfunctions with respect to different eigenvalues reflect structural features at different scales, which has been utilized in surface segmentation and reconstruction [14,17]. The eigenfunctions vary along the object surface and are invariant to different poses, which makes them ideal for describing the structural feature of the object. A variety of applications have been introduced taking the advantages of eigenfunctions, such as pose-invariant Reeb graph [16], shape matching [13] or registration [18], and the Shape-DNA [15]. Another important application of eigenfunctions is surface quadrangulation or parameterization [19]. For example, the Morse-Smale complexes [20–22] were built by connecting the saddle and extrema of eigenfunctions, dividing the surface into several coarse quadrilateral

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patches. Despite of these developments, feature alignment is still a challenging problem in surface parameterization.

In this paper, we introduce a novel method to define a guidance for cross field generation using eigenfunctions, and generate a structure-aligned parameterization for the input triangle mesh. A guidance is first constructed using the gradient of multiple eigenfunctions of the LBO to capture the structural feature at various scales. Then a smooth cross field is built following the guidance, based on which a surface parameterization is computed. The main contributions of our work include:

1. A novel structure-aligned approach is developed for surface parameterization using eigenfunctions, which is insensitive to local detailed surface features.
2. Multiscale structural features are captured using the gradient of multiple eigenfunctions as a guidance for cross field generation.
3. A new algorithm is introduced to enable anisotropy in the parameterization by adapting the cross field to non-uniform parametric line spacings.

The remainder of the paper is organized as follows. Section 2 describes eigenfunctions. Section 3 explains how to define the guidance for cross field construction using the gradient of multiple eigenfunctions. Section 4 discusses cross field construction and surface parameterization. Section 5 shows some results. Finally, Section 6 draws conclusions and points out future work.

2. Eigenfunctions

Given a G^2 smooth surface S, the eigenproblem of LBO is to find the eigenvalues λ and their corresponding eigenfunctions f defined on it, such that

$$-\Delta_S f = \lambda f, \tag{1}$$

where Δ_S is the LBO defined on surface S. Since $-\Delta_S$ is a self-adjoint and semi-positive definite operator, the eigenvalues of $-\Delta_S$ are real and nonnegative. Eigenfunctions of the LBO provide a set of convenient basis to describe the shape behavior or structural feature of an object. Let M be a triangulation of surface S, $\{\mathbf{x}\}_{i=1}^{n}$ be the vertex set of M. A class of discretization scheme [23–25] for the LBO can be represented as

$$\Delta_S f(\mathbf{x}_i) \approx \sum_{j\in N(i)} w_{ij} f(\mathbf{x}_j), \quad w_{ij} \in \mathbf{R}, \tag{2}$$

where $N(i)$ contains the 1-ring neighborhood of $\mathbf{x}_i$, and w_{ij} are the weights defined in different discretizations of the LBO. The eigenproblem becomes

$$-\sum_{j\in N(i)} w_{ij} f(\mathbf{x}_j) = \lambda f(\mathbf{x}_i), \tag{3}$$

or in matrix form,

$$-WF = \lambda F, \tag{4}$$

where $F = [f(\mathbf{x_1}), \ldots, f(\mathbf{x_n})]^T$ and W is the coefficient matrix defined by w_{ij}. Eq. (4) yields n *modes*, each corresponds to an eigenvalue and eigenfunction. Let λ_k and F_k $(k = 0, 1, \ldots, n-1)$ be the kth eigenvalue and the corresponding eigenfunction, we have

$$0 = \lambda_0 \leqslant \lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_{n-1}. \tag{5}$$

$\lambda_0 = 0$ represents a rigid-body mode, its eigenfunction F_0 is a constant-scalar field.

Various methods have been developed for the discretization of LBO. In this paper, we use the cotangent scheme [13,26–29]. This discretization provides a symmetric matrix, which makes all the resulting eigenvalues and eigenfunctions real. However, the cotangent scheme was proved to be not convergent for irregular nodes, and it cannot deal with non-uniform meshes well [25]. There are some research conducted on convergent discretization of the LBO. For example, a k-nearest neighbor of a vertex was considered using a truncated heat kernel [30]. In [25], the Laplace matrix was constructed based on a quadratic fitting and its convergence rate was proved to be linear [31]. This discretization provides a non-symmetric matrix, resulting in complex eigenvalues and eigenfunctions. In this paper we use the cotangent scheme LBO to obtain real eigenfunctions.

3. Guidance estimation using eigenfunctions

Different eigenfunctions reflect surface features at different scales [14]. Compared with the high-mode eigenfunctions used in [20–22], the low-mode eigenfunctions are less sensitive to the detailed surface features and capture the major structure of the object. In this paper, we will use multiple low-mode eigenfunctions to design a direction guidance and then build a cross field, from which we can obtain a structure-aligned surface parameterization.

The gradient of the eigenfunctions can be used to represent structural features. For example in the Hand model in Fig. 1, the gradient of the first and second eigenfunctions (black arrows) reflects the slim cylindrical structure of the fingers. However, a single eigenfunction may only reflect features in certain regions well. For example in (b), the gradient of Mode 1 eigenfunction follows the middle finger and the thumb very well, but not the little finger because the gradient magnitude is very small on it. Similarly in (c), the gradient of Mode 2 eigenfunction follows the index, third and little fingers well but not the thumb. From Fig. 1, we can observe that each eigenfunction plays a dominant role in certain regions, where the gradient of this eigenfunction reflects the structural features at a certain scale. We call such region a *feature region* of that eigenfunction. By combining the gradient in the feature regions from multiple eigenfunctions, we can build a structure-aligned guidance for the cross field construction. For example, we can define the middle finger and the thumb as the feature region of Mode 1 and the index, third and little finger as the feature region of Mode 2. Then the slim cylindrical structure of all five fingers can be captured using these two modes.

Then, the next problem is how to represent the feature region for each eigenfunction. Here, we design two different ways to represent the feature regions: isocontours and characteristic values.