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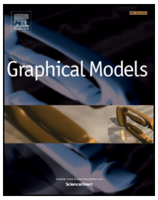

An iterative predictor–corrector approach for modeling static and kinetic friction in interactive simulations

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ABSTRACT

In this paper we propose a novel iterative predictor–corrector (IPC) approach to model static and kinetic friction during interactions with deformable objects. The proposed IPC method works within the purview of the implicit mixed linear complementarity problem (MLCP) formulation of collision response. In IPC, first the potential directions of frictional force are determined at each contact point by leveraging the monotonic convergence of an iterative MLCP solver. All the contacts are then categorized into either static or kinetic frictional states. Linear projection constraints (LPCs) are used to enforce 'stiction' for contacts in static friction. We propose a *modified iterative constraint anticipation* (MICA) approach that can resolve the LPCs while simultaneously solving the MLCP. Our method can handle arbitrary models including asymmetric and anisotropic friction models. IPC requires low memory and is highly tunable. Multiple example problems are solved to demonstrate the method.

1. Introduction

Friction forces oppose relative motion or impending movement between surfaces in contact and can play a substantial role in altering the overall dynamical behavior of a mechanical system. *Dry friction* comes into play when two solid surfaces are in contact. This can either be *static*, i.e., between surfaces not in relative motion or *kinetic*, i.e., between surfaces in relative motion. In this paper, we present a unified approach to simulate the *Coulomb* model of dry friction between a rigid and one or more deformable objects in interactive simulations.

Interactive simulation is particularly demanding in terms of computational speed. For real time graphics, a minimum update rate of 30 Hz is necessary, whereas a much higher update rate of up to 1 kHz may be necessary for stable force (haptic) feedback [1]. Physics-based methods are often used to model real world objects in interactive simulations. Contact and friction are essential components of such simulations. While modeling Coulomb dry friction for deformable objects, in addition to position/velocity and contact forces, one is required to determine (a) friction states: *static* or *kinetic*, (b) direction of frictional force and (c) its magnitude.

In contrast to methods used in more traditional areas like computational mechanics for simulating the same phenomenon, there is a greater variability in methods used in interactive simulations. In interactive simulations, there is greater emphasis on speed as compared to computational mechanics where physical accuracy (closer to first principles) is paramount. Algorithms employed to find these additional unknowns of the friction model are linked to the underlying methods used to update deformations and to compute collision response. Physical accuracy is often sacrificed for simulation speed and vice versa. With recent advances in computer architecture, it has become increasingly possible to employ physically accurate methods such as finite elements while still achieving the goals related to speed and

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perceptibility [2–5]. The challenge is to develop algorithms that are both physically accurate and are suitable for VR related applications.

Lumped mass systems models like implicit mass-spring models use an impulse-based approach to evaluate unknowns of the friction problem. Each mass under frictional contact is analyzed after updating positions/velocities and forces at each time step with a frictional impulse to categorize the contacts into static and kinetic frictional contacts. Appropriate forces are then applied in the next time step to simulate the effect of friction. This approach however can lead to non-physical and often unstable behavior and is mostly employed in mass-spring based formulations.

A physically more accurate way to model Coulomb friction is by using a linearized friction cone approximation [6,7] which results in a mixed linear complementarity (MLCP). In this approach the friction cone, non-linear by nature, is linearized using a polyhedral pyramid approximation (see Fig. 3) leading to an MLCP. The MLCP formulation involves a system of linear inequalities accompanied by orthogonality conditions. This approach is commonly employed in modeling friction for rigid body simulations [8,9]. In recent years finite element (FE) based methods that are common in engineering, are gaining wide-spread use in graphics applications. FE equations are derived from partial differential equations of motion and hence offer greater physical accuracy over lumped parameter models. Implicit contact resolution using FE-based MLCP formulation offers greater physically accuracy. The linearized friction cone approach can lead to physically accurate simulations compared to methods employing frictional impulses. However, this approach introduces modeling complexity adding substantial number of unknowns to the system thereby requiring considerably higher memory and additional book-keeping. Besides, an extension to simulate arbitrary frictional models is not straightforward.

In this paper, we introduce a novel iterative predictor–corrector (IPC) method to simulate Coulomb friction within the MLCP framework for deformable objects in contact with a solid object. IPC method (a) avoids otherwise large systems of MLCP (b) allows arbitrary friction models (c) reduces bookkeeping and memory footprint and (d) is individually (just friction model) tunable. IPC avoids approximation of the friction cone but instead works at the solver level to determine the unknowns of a frictional contact which include the friction force, its direction and the frictional state. We devise an approach that uses the knowledge of intermediate solution of the MLCP solver to determine the unknowns of a frictional contact. We also proposed a modification of this solver to enforce no slip conditions between surfaces in static friction. IPC works with both FE and mass-spring based implicit MLCP formulations of both cloth and 3D objects with arbitrary (including anisotropic) friction models. In addition, IPC eliminates the need for large MLCPs, consumes very low additional memory and requires no additional bookkeeping.

2. Related work

Coulomb model of dry friction is the most commonly used model due to its simplicity and its ability to model most frictional effects that are commonly observable in daily life. Various methods have been proposed in literature to simulate Coulomb friction for deformable body simulations in virtual environments. Most of the existing methods are designed to work with a specific underlying formulation for deformation and the collision response employed.

Methods used to simulate isotropic Coulomb friction in mass-spring based simulations analyze the change in tangential velocity when limiting frictional forces are applied to categorize contacts into static and kinetic frictional states. In [10], Bridson et al. provide a well-known approach to robustly treat collisions and evaluate impulses from frictional forces for mass-spring based cloth simulations. In this approach the normal force computed from the collision response algorithm is used to determine a maximum frictional impulse (limiting friction) which when applied to a nodal mass point under contact results in a change of its tangential velocity. If this change in velocity is larger than the 'pre-friction' velocity, then the static friction case is enforced by making the tangential velocity zero. Otherwise the node is allowed to slide under kinetic friction. This method was later improved by Selle et al. in [11]. In [12] Harmon et al. builds upon the approach proposed in [10] to allow for improved estimation of tangential sliding contact velocities. The global velocity vectors are decomposed into sliding and normal velocities. The sliding velocities are then used to determine the change in momentum due to friction.

In [13], Pabst et al. proposed a method to model anisotropic frictional effects in the impulse response framework of mass-spring based simulation using friction tensors. Friction tensor maps tangential motion to forces in the tangent plane and can be designed to simulate asymmetric and heterogeneous friction effects. Static and kinetic friction effects are simulated using the same approach proposed in [10]. Chen et al. [14] propose a non-linear anisotropic friction model and further improved the quality of simulations by obtaining parameters of the non-linear model from the experiments.

Impulse-based approaches described above cannot be employed in the case of finite element-based simulations but instead a linearized friction cone model, now common in rigid body simulations, is used. Pang et al. introduced this for rigid body simulations (refer [15]) in which a MLCP based model to friction has been formulated. In this approach MLCP is converted to LCP by *static condensation*. Otaduy et al. modeled friction through a similar approach for contacts between deformable objects [6]. However, the computational complexity involved in inverting the system matrix during static condensation in the case of deformable bodies is prohibitive due to presence of off-diagonal components. This procedure was avoided by Otaduy et al. through a novel *iterative constraint anticipation* (ICA) method to solve the MLCP. This solver works by decoupling the solution of Lagrange multipliers/frictional forces with the body positions/velocities. Frictional forces are modeled using a 4-sided friction cone. However, in order to modify the resulting MLCP friction from friction cone linearization to be in the form suitable for ICA method, the unknowns of the 4-sided friction cone are compacted to two that align with the local contact co-ordinates. This is possible by assuming the frictional direction to be along the contact sliding velocity from the previous time