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Smooth convolution-based distance functions

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ABSTRACT

Smooth surface approximation is an important problem in many applications. We consider an implicit surface description which has many well known properties, such as being well suited to perform collision detection. We describe a method to smooth a triangle mesh by constructing an implicit convolution-based surface. Both the convolution kernel and the implicitization of the mesh are linearized. We employ the straight skeleton to linearize the latter. The resulting implicit function is globally C^2 continuous, even for non-surface points, and can be explicitly analytically evaluated. This allows the function to be used in simulation systems requiring C^2 continuity, for which we give an example from industrial simulation, in contrast to methods which only locally smooth the surface itself.

1. Introduction

In many applications smooth surface approximation of triangular networks is an important problem. Variational design [16] is a standard technique to solve these kinds of problems. Usually B-spline surfaces are targeted. Well known methods dealing with explicit surface representations, such as corner cutting, lead to B-spline patches or more generally subdivision surfaces [11]. Other methods employ fairing of the meshes while still keeping a mesh structure [13]. Another possible way to represent surfaces is as implicit functions. Implicit surfaces have proven to be a powerful tool in surface design [9,26,29]. When the surfaces represent the boundaries of solids, it is often important to efficiently determine the inside and outside of the solids. This can easily be achieved with implicit surfaces using a sign check [25].

Our goal is to get a smooth approximation of the polygonal mesh while still being able to control the original mesh interpolating surface. Therefore we introduce a feature size parameter that controls the smoothing effect. Another constraint on our implicit representation, motivated by the application presented in Section 7, is being able to compute gradients and second derivatives and to perform collision detections (inside/outside checks) for non-surface points.

We certainly could create smooth surfaces with rather good approximation quality using subdivision surfaces, but as we aim to use an implicit surface description, a global implicitization would require an expensive computation. The smoothing effect would also be local and would affect only a small neighborhood of the surface (cf. [21] Section 2.3).

The main contribution of this paper therefore is presenting a convolution-based implicit surface definition for triangle meshes, whose underlying distance function is globally smooth even on non-surface points. We employ linearizations that allow the function to be computed analytically. The linear kernel used is not novel, but to our knowledge its use in this context is. Notable prior work are Bloomenthal's convolution surfaces [8] and Colburn's smoothing method [12]. Differences to our approach are detailed below.

Whenever it aids visualization or discussion, figures show the equivalent 2-d representation of our method. Fully 3-d

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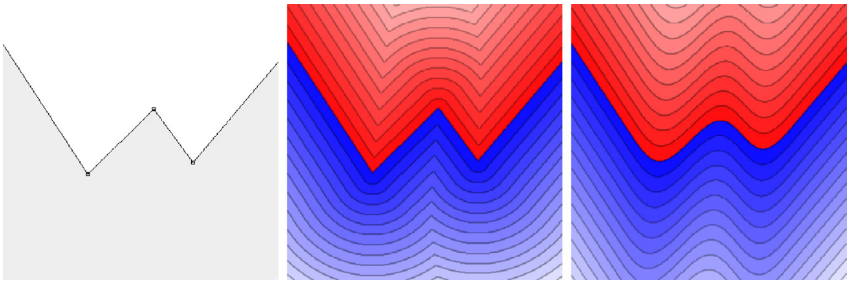

Fig. 1. A solid with sharp edges (left), its signed minimum distance field (middle), and our smoothed distance field (right).

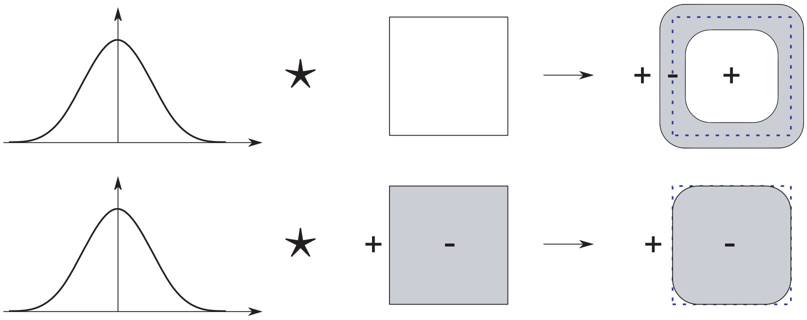

Fig. 2. Convolution with skeleton (top) and solid (bottom).

images are given where necessary, see also Section 6. We also concentrate on triangle meshes only, since polygon meshes can be converted to triangle meshes easily. Fig. 1 shows the result of our method applied to a solid in 2-d. While the signed minimum distance function has cusps where it is not differentiable, our smoothed distance field is C^2 everywhere. This cannot be achieved by only smoothing out the solid's edges and then computing the signed minimum distance, as this would still lead to cusps not lying on the surface.

This paper is structured as follows: in Section 2 we give a brief survey of implicit functions and describe a convolution-based smoothing approach. Our choice for a linearized kernel is described in detail in Section 3. In Section 4 we give a brief survey of the straight skeleton as described by Aichholzer et al. and use it to give an implicit surface definition. We discuss the approximation quality of the resulting surface in Section 5, give results of smoothed surfaces in Section 6, and present an application in Section 7.

2. Implicit surfaces

Implicit functions are widely used tools to describe surfaces and solids in geometric modeling. In the most general form, an implicit surface is the iso-surface of a potential field F for an iso-value T:

$$F(\mathbf{x}) - T = 0. \tag{1}$$

Solids can be described as the interior of an implicit surface, i.e. all points $\mathbf{x}$ for which $F(\mathbf{x}) - T \leq 0$. CSG operations, such as union and intersection of different solids, can then be implemented easily as min and max operations.

Convolution surfaces, as first described by Bloomenthal and Shoemake [8], are implicit surfaces based on skeleton primitives like lines and triangles. The resulting surfaces are a smooth blend of iso-potentials around the primitives, but do not approximate the original mesh in the way we intend. The convolution integrals for these surfaces are mostly computed by a numerical scheme. A different approach is Colburn's method [12], who does corner smoothing by a numerical convolution of a solid's characteristic function with a Gaussian function. Our method employs a linearization of the convolution kernel as well as an implicit description of the triangle-manifold. This allows us to exactly evaluate the implicit function without reverting to numerical methods, in contrast to the numerical integration needed for Colburn's approach.

Bloomenthal and Shoemake describe convolution surfaces as an extension of Blinn's blobby model [7] for implicit surface design. They employ a convolution of a filter kernel over a skeleton to create a surface. The resulting surfaces resemble the skeleton, but surround it instead of approximating it. Fig. 2 schematically illustrates the conceptual difference between convolution surfaces on the one hand and the surfaces created by Colburn's method and ours on the other hand. Bloomenthal's convolution surfaces integrate over skeletons only, whereas we integrate over the complete domain of the implicit function. While both methods nicely round the corners of the square, convolution surfaces create an exterior and interior surface offsetted from the original skeleton. The innermost area has the same sign as the outside area. Thus, in our scenario convolution surfaces cannot simply be applied to a solid's boundary, because, without further computation, they do not lead to a signed distance function to discriminate inside and outside.

To apply a convolution-based smoothing, we first have to convert the explicitly given triangle mesh to an implicit