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The metric of colour space

Jens Gravesen

Department of Applied Mathematics and Computer Science, Technical University of Denmark, Richard Petersens Plads Bygning 324 DK-2800 Kgs. Lyngby, Denmark

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ABSTRACT

The space of colours is a fascinating space. It is a real vector space, but no matter what inner product you put on the space the resulting Euclidean distance does not correspond to human perception of difference between colours.

In 1942 MacAdam performed the first experiments on colour matching and found the MacAdam ellipses which are often interpreted as defining the metric tensor at their centres. An important question is whether it is possible to define colour coordinates such that the Euclidean distance in these coordinates correspond to human perception.

Using cubic splines to represent the colour coordinates and an optimisation approach we find new colour coordinates that make the MacAdam ellipses closer to uniform circles than the existing standards.

1. Introduction and background

The human retina has three types of colour photo receptor cone cells, with different spectral sensitivities, see Fig. 1, resulting in trichromatic colour vision, i.e., a colour is described by three real numbers. A fourth type of photo receptor cells, the rod, is also present, but it is only used at extremely low light levels (night vision), and does not contribute to the perception of colour. The sensitivities of the colour receptors are not the same for all humans. It depends on the angle under which the colour is observed, but also on age and gender and there are individual variations. Furthermore the perception of colour depends not only on the stimuli of the colour receptors, but also on the environment. The spectral distribution of light reflected from a piece of paper will depend on the light that hits the paper. So if we compare daylight in the morning, daylight at noon, and indoor lightning we get very different spectral distribution, but we will in all cases perceive the reflected light as the same white colour. There are also memory effects: if you watch a colour image which is instantaneously replaced with a grey image you will for a short while perceive not the grey image, but a colour image consisting of the complementary colours. So there is more to colour perception than the light that hits the eye. We will not consider this very complicated processing done by the brain, but only consider the results of colour perception experiments that have been conducted under very controlled conditions.

In an experiment in 1942 MacAdam discovered that human perception of distance between colours does not correspond to any Euclidean distance in colour space, [9]. These experiments have been repeated and extended many times, see [12]. The results of the experiments are reported as ellipses in 2D and ellipsoids in 3D that can be interpreted as geodesic spheres of a fixed radius. There have since been many attempts to find a distance on colour space that corresponds to human perception. One way is to define new coordinates on colour space such that the Euclidean distance between these coordinates corresponds better to human perception. Most noticeable are the CIE76 standard using the CIE Luv or Lab coordinates, [12], the CIE94 standard using the CIE LCh coordinates, [3], and latest the CIEDE2000

E-mail address: jgra@dtu.dk

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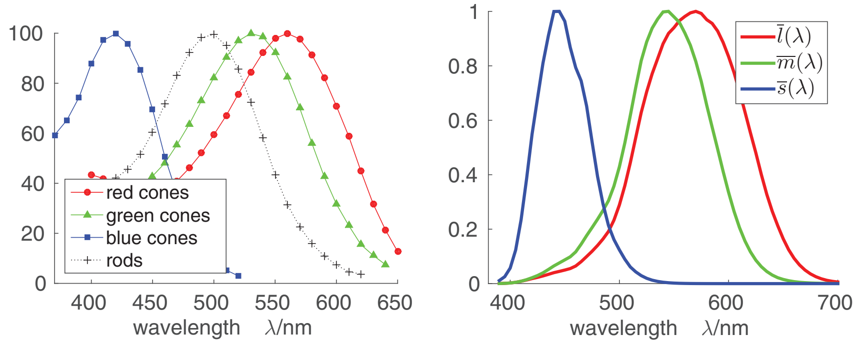

Fig. 1. To the left the normalised absorbance spectra of the four human photo receptors, [6]. To the right the normalised sensitivity of the three colour receptors of the human eye, according to the CIE 2006 physiological model, [5], (age 32, angle 2°). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

standard, [4,11]. The CMC l:c standard (1984) also used LCh coordinates; it is a British Standard (BS 6923:1988).

The definition and parametrisation of colour space is an old problem that has attracted interest from many scientists, including names such as Helmholtz and Schrödinger. A recent paper [8] uses a grid optimisation approach to find colour spaces with better perceptual uniformity. Besides perceptual uniformity it is also required that the colour attributes lightness, chroma, and hue are easily obtained. Another difference is that they want the Euclidean distance between colours to agree with the standardised colour-difference formulas above, while we want to improve on those.

In Section 2 we present the basic terminology and colour theory, in particular the classic colour coordinates. In Section 3 we present the problem of defining a colour difference and describe some existing standards.

In Section 4 we present a method that only focuses on perceptual uniformity and gives us good coordinates on colour space. We consider colour space as a Riemannian manifold and perfect coordinates would give an isometry to Euclidean space. Due to lack of data in electronic form we will only consider MacAdam's original results. MacAdam's experiments took place in a two dimensional slice of colour space so we will only consider the 2D case where luminance is constant. The procedure is a simple two stage process:

1. We identify the MacAdam ellipses with a metric at each of their centres and extend those to a Riemannian metric on all of colour space.
2. We determine a near isometry to Euclidean space $\mathbb{R}^2$.

We use cubic B-splines to represent both the Riemannian metric and the map to $\mathbb{R}^2$ and the two steps can be performed by solving a quadratic optimisation problem. Even though we only consider the 2D case the method is general and can be extended to full 3D colour space.

The result of this process depends on how well the chosen splines can approximate the solution to the two optimisation problems. We expect better results if we increase the degree and/or refine the knot vectors. In this work we have used cubic splines and refined the knot vector until a further refinement did not change the result noticeably. We expect the cubic spline to be close to the true optimum and that we will not obtain any significant improvement by raising the degree of the splines.

Step one is the crucial step. As soon as the Riemannian metric is chosen the best near isometry is essentially fixed. The only freedom left is how to measure the distance from being an isometry. We have used some kind of L^2 distance, but one could of course also use L^1, L^∞, or other distances.

In step one we have chosen the interpolant by simply minimising the second derivative of the components of the logarithm of the metric tensor. This leads to a quadratic optimisation problem but perhaps it would be better to minimise the second derivative of the components of the metric tensor. It would also be possible to consider the curvature of the space and ask for it to be as constant as possible or perhaps as close to zero as possible. Determining the best approach requires more research, should be done using all the available data, not just the classical MacAdam ellipses, and ideally in collaboration with colour scientists.

One can argue that the MacAdam ellipses do not determine the metric at their centres but rather are the geodesic unit circles. This point of view leads to a novel geometric question, namely to what extent the unit spheres of a Riemannian manifold determine the metric. We make this precise in Section 6. We finally conclude in Section 7.

2. Colour space

The International Commission on Illumination, (CIE[1]) has defined several parametrisations of the space of colours, but the starting point and the coordinates in which most experiments are reported are the *CIE XYZ components*. If $I : [\lambda_1, \lambda_2] \to \mathbb{R}_+$ is the intensity function for the light, then the CIE XYZ components are defined by

$$(X, Y, Z) = k \int_{\lambda_1}^{\lambda_2} I(\lambda) \left(\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)\right) \mathrm{d}\lambda\,, \tag{1}$$

where the functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are the CIE 1931 Standard Colourimetric Observers, see Fig. 2, and k is a normalisation constant which makes $Y = 100$ for a standard

[1] http://www.cie.co.at.