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Generalization of the incenter subdivision scheme ☆,☆☆

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ABSTRACT

We introduce a new interpolatory subdivision scheme generalizing the incenter subdivision [8]. The proposed scheme is equipped with a shape controlling tension parameter, is Hermitian, and reproduces circles from non-uniform samples. We prove that for any value of the free parameter the limit curve is G^1 continuous. The scheme is shape preserving and avoids undesirable oscillations by producing curves with a finite number of inflection points at the regions where the control polygon suggests a change of convexity. Several examples are presented demonstrating the properties of the scheme.

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1. Introduction

Due to their simplicity and flexibility, subdivision methods have become a very popular choice for CAGD and related application areas. Starting with an initial control polygon, curve subdivision algorithms are iterative methods refining the polygon according to prescribed rules. The main challenge regarding the design of subdivision algorithms is to devise subdivision rules such that the iterative process converges, the limit curve is smooth and the scheme has attractive properties such as the preservation of the convexity of the initial polygon.

Interpolatory subdivision schemes in particular have attracted considerable research interest since many applications require the generation of curves passing through the vertices of the initial control polygon. The interpolation of the initial data is usually achieved by using subdivision rules that at any step keep all the vertices of the previous step.

Inspired by the incenter subdivision recently introduced in [8], in this paper we propose a family of interpolatory Hermite subdivision schemes with a free parameter. As in [8], the new point splitting a given edge is inside the triangle defined by that edge and the tangent lines at its two ends. The free parameter $0 < \rho < 1$ controls the position of the new point, which coincides with the incenter of the triangle for $\rho = 0.5$. Nevertheless, the subdivision curve of our scheme is different from the one obtained in [8], even if we take $\rho = 0.5$, since our scheme preserves the initial tangent vectors, while the scheme in [8] updates the tangent vectors at each subdivision step.

The proposed subdivision scheme is G^1 continuous for any value of the free parameter ρ. For $\rho = 0.5$, the discrete curvatures at the vertices of the control polygons converge, exhibiting a behavior close to continuity. Moreover, the scheme preserves the shape of the initial polygon, avoids undesirable oscillations of the limit curve and introduces inflection points only in those regions of the curve where the control polygon suggests a change of convexity. Furthermore, if the vertices of the initial polygon and the associated tangent vectors are sampled from a circle, not necessarily uniformly, then the limit subdivision curve is a perfect circle.

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Relevance: The results of the tests and experiments in [8] show that the limit curves of the incenter subdivision are high quality, visually pleasing shapes, even for challenging initial control polygons where stationary linear schemes produce artifacts. The generalized incenter subdivision has further advantages which might be significant in practical applications.

Firstly, it gives to the user a shape controlling tension parameter. Notice that this can be a very desirable property in certain design applications, even when this extra freedom cannot be exploited to increase the smoothness of the curve. Notice also that the original incenter subdivision does have user-defined parameters, which are related to the recomputation of the tangents at subdivision step. However, the tests in [8] show that these parameters do not affect the global shape of the limit curve significantly.

Secondly, the generalized scheme is Hermitian. In contrast, in the incenter subdivision, the tangents of the initial control polygon affect the shape of the limit curve, however, the relation between the tangents of the initial polygon and the corresponding tangents of the limit curve is not clear.

Finally, the theoretical results of Section 3.3 proves that scheme proposed in this paper for $\rho = 0.5$ shows a behavior of the discrete curvature that is close to be continuous, even when consecutive edges have a big difference in their lengths or when they form a sharp corner.

1.1. Related work

Hermite subdivision schemes are a particularly expressive tool for curve design since they take into account not only the vertex positions, but also tangent vectors prescribed at the vertices. They were introduced in [11] where a family of Hermite interpolatory schemes with two parameters producing C^1 continuous functions was proposed. Merrien's family includes as a special case the C^1 cubic spline interpolating prescribed values and first derivatives. To study interpolatory Hermite subdivision schemes, Dyn and Levin [5,6] consider an associated point subdivision scheme generating the divided differences of the vectors of the function values and the corresponding derivatives. In [9], a different approach is proposed using splines that interpolate the values and derivatives of functions.

The incenter scheme in [8] and the generalization proposed here are examples of geometrically controlled subdivision schemes where new points are computed by geometric constructions on the control polygon.

In [10] several generalizations of the classical 4-points subdivision scheme [7] are introduced. In these schemes a geometric construction is used to compute local values for the tension parameters which allow to obtain convexity preserving and artifact free subdivision schemes. As the new points are constructed indirectly, that is, through the computation of local values for the tension parameter, the search space for the insertion of new points is limited to a line, and thus, the properties of the schemes are not always optimal. Moreover, rigorous proofs of the C^1 continuity of the displacement-safe schemes a and co-convex scheme are missing and the circle reproduction property is not addressed in any of the schemes presented in [10].

In [15], a geometrically controlled Hermitian subdivision scheme is proposed. Instead of splitting the angles between the tangents defined at the vertices and the edges as is the case in [8], in an approach more similar to [10], a new point splits the old edge at a chosen ratio and then a displacement vector is computed as a linear combination of the normals defined at the ends of the old edge. Normal vectors at selected vertices can be interpolated and the subdivision curve is shape preserving. Nevertheless, it shows abrupt changes of the discrete curvature (see Fig. 7 in [15]) when the initial polygon has sharp corners or the length of consecutive edges is very different. Furthermore, the performance of the discrete curvature is not addressed in any case.

Recently proposed geometric subdivision algorithms producing curves with properties similar to the incenter scheme and its generalizations include [2,3,14], which also reproduce circles and generate G^1 continuous subdivision curves. In [12], the limit curve is C^2 continuous, but circles are reproduced only if the vertices of the initial polygon are uniformly distributed. In [13], a Hermite interpolatory subdivision scheme is presented which requires first and second derivative vectors associated with the vertices of the control polygon. The scheme depends on a tension parameter and the limit curve is C^2 continuous if the tension parameter is set to 1. Moreover, it reproduces a perfect circle from arbitrary sequences of points sampled from that circle.

1.2. Overview

The rest of the paper is organized as follows. In Section 2, we introduce the notation, define the subdivision scheme and derive some of its basic properties. In Section 3, we prove that the subdivision scheme converges, the limit curve is G^1 and that discrete curvatures at the control points converge. In Section 4, we show several examples and numerical experiments and we briefly conclude in Section 5.

2. The subdivision scheme

Let $P_i^0 \in \mathbb{R}^2,\ i = 0, \ldots, n$ be the vertices of the initial polygon enumerated in clockwise order and denote by $\mathbf{t}_i^0$ the normalized tangent vector associated to the vertex P_i^0. We assume that

- the initial polygon does not contain three consecutive collinear points,
- tangent vector $\mathbf{t}_i^0$ is contained in the angular sector defined by $P_{i+1}^0 - P_i^0$ and $P_i^0 - P_{i-1}^0$ and
- for all index i, the angle α_i^0 from $P_{i+1}^k - P_i^k$ to $\mathbf{t}_i^0$ and the angle β_{i+1}^0 from $\mathbf{t}_{i+1}^0$ to $P_{i+1}^k - P_i^k$ are smaller than $\frac{\pi}{2}$.

The last two hypothesis will be used to obtain a shape preserving subdivision curve. Tangent vectors may be defined by the user or computed automatically, see [1] and also Section 4.

2.1. Preprocessing

With slight differences, our notation is similar to [8] to make the comprehension easier for the reader. We denote

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