Graphical Models 71 (2009) 107–112

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

Computing the minimum distance between a point and a clamped B-spline surface

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A R T I C L E   I N F O

Article history:
Received 10 February 2008
Received in revised form 28 May 2008
Accepted 9 January 2009
Available online 24 January 2009

Keywords:
Minimum distance
B-spline surface
Newton's method
Exclusion criterion

A B S T R A C T

The computation of the minimum distance between a point and a surface is important for the applications such as CAD/CAM, NC verification, robotics and computer graphics. This paper presents a spherical clipping method to compute the minimum distance between a point and a clamped B-spline surface. The surface patches outside the clipping sphere which do not contain the nearest point are eliminated. Another exclusion criterion whether the nearest point is on the boundary curves of the surface is employed, which is proved to be superior to previous comparable criteria. Examples are also shown to illustrate efficiency and correctness of the new method.

1. Introduction

The computation problem of the minimum distance between a point and a surface is also called the point projection problem of the surface. The distance information between a point and a surface is very important for interactively selecting surfaces and surface construction in geometric modeling [3,5], for collision detection and physical simulation in computer graphics and computer vision, for interference avoidance in CAD/CAM and NC verification [4,8]. It is also essential for the B-spline surface fitting problem [8,12].

Suppose that a surface has a parametric form $\mathbf{S}(u,v)$, and that the squared distance from $\mathbf{S}(u,v)$ to a test point $\mathbf{p}$ is $H(u,v) = (\mathbf{S}(u,v) - \mathbf{p})^2$. Then the following system of equations

$$\begin{cases} H_u(u,v) = (\mathbf{S}(u,v) - \mathbf{p}) \cdot \mathbf{S}_u = 0 \\ H_v(u,v) = (\mathbf{S}(u,v) - \mathbf{p}) \cdot \mathbf{S}_v = 0 \end{cases} \qquad (1)$$

is satisfied at the nearest point on the surface $\mathbf{S}(u,v)$ to the test point $\mathbf{p}$. Thus, the point projection problem is turned into a root-finding problem of the equation system (1) [2,7,10,13]. There are two main steps in the above methods. The first step is to remove the regions or patches which contain no solutions, and the second one is to compute local extrema with numerical methods such as the Newton method. Zhou et al. use both the Projected-Polyhedron and Linear Programming methods for the first step [13], while Johnson utilizes the tangent cone method instead [2].

However, not all of the solutions of the equation system (1) need to be computed. In the worst case, as shown in Fig. 1, the nearest point $\mathbf{q}$ is a corner control point of the surface, and it is not mapped to any solution of the equation system (1). Thus, all of the computation on solving the solutions of the equation system (1) is unnecessary.

Recently, instead of analyzing the solutions of the equation system (1), several geometric methods combining elimination technique with subdivision technique are used for the point projection problem of NURBS surfaces [5,8,11]. In these methods, the surface may be iteratively subdivided into surface patches, and most of the surface

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doi:10.1016/j.gmod.2009.01.001

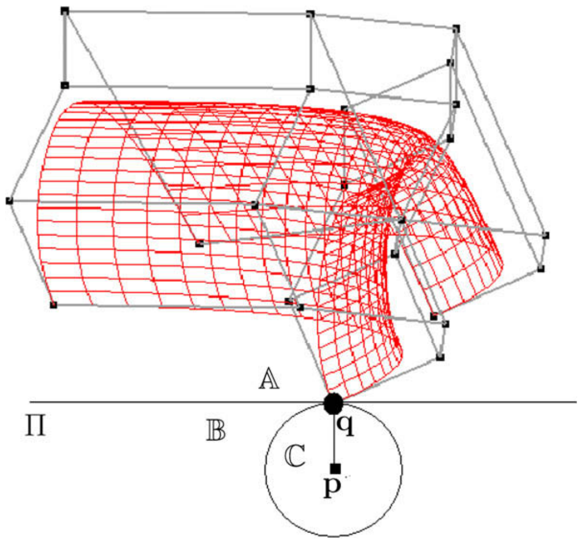

Fig. 1. An example shows that the nearest point is not the solution of Eq. (1). **p** is a test point, and the nearest point **q** is one of the corner points of the surface. All the other control points are above the plane Π, which passes through **q** and is perpendicular to the direction vector from **p** to **q**.

patches not containing the nearest point are eliminated by analyzing the relationship between these surface patches and the test point. When the remaining surface patches are small or flat enough, then the Newton's method is used to solve the nearest point for each remaining patch. The subdivision process of the NURBS surfaces is robust, and these methods seem geometrically more intuitive, reasonable and robust. The key technique of these geometric methods is how to eliminate the removable surface patches. Piegl and Tiller directly decompose the base surface into quadrilaterals within a relative geometric tolerance [8]. However, such a decomposition process is pointed out to be time-consuming [5]. To reduce the computation time of decomposition, Ma and Hewitt present a control polygon approach for the same problem [5]. They provide a condition that the nearest point is on one of the four boundary curves. Selimovic provides an improved algorithm by using two exclusion criteria [11]. The first one is a sufficient condition that the nearest point is one of the four corner control points, and the second one is another sufficient condition that the nearest point is on one of the four boundary curves. For the case shown in Fig. 1, the method of [11] prunes the surface patch directly, and costs no computation on solving the solutions of Eq. (1).

This paper presents a spherical clipping method for the point projection problem of clamped B-spline surfaces. A clipping sphere with its center point being the test point **p** is introduced, and the radius of the sphere is set by the Euclidean distance between the test point **p** and a point of the surface. Thus, any surface patch outside the clipping sphere can be eliminated. Another exclusion criterion is also employed by using a new sufficient condition whether the nearest point is on one of the boundaries curves of the surface. The algorithms in [5] have an overlooked flaw and may fail in some cases [1]. For clamped B-spline surfaces, we will prove that the criterion in our algorithm is superior to the comparable criterion in [11].

We assume that the clamped B-spline surface is defined by

$$\mathbf{S}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} B_{i,p}(u)B_{j,q}(v)\mathbf{P}_{i,j}, \quad u \in [u_0, u_r],\ v \in [v_0, v_k], \tag{2}$$

where u and v are the parameters of the surface, p and q are the degrees of the surface in u and v directions, $\{\mathbf{P}_{i,j}\}$ are the control points in the space $\mathbb{R}^3$, $\{B_{i,p}(u)\}$ and $\{B_{j,q}(v)\}$ are the pth-degree and the qth-degree B-spline basis functions based on the knot vectors $U = (u_0, \dots, u_0, u_1, \dots, u_{r-1}, u_r, \dots, u_r)$ and $V = (v_0, \dots, v_0, v_1, \dots, v_1, \dots, v_k, \dots, v_k)$, respectively.

This paper is organized as follows. Section 2 presents the outline of the new algorithm. Section 3 compares the new method with the method in [11]. Analysis and examples are also shown in this section. Some conclusions are drawn at the end of this paper.

2. Algorithm for the point projection problem for B-spline surfaces

The spherical clipping method is explained in this section. The basic idea is as follows. Suppose that the point **p** is a test point, the point **q** is a point on the surface, and $\mathbf{O}(\mathbf{p}, ||\mathbf{pq}||)$ is a sphere with the center point **p** and its radius $||\mathbf{pq}||$. The nearest point to the point **p** must be inside the sphere $\mathbf{O}(\mathbf{p}, ||\mathbf{pq}||)$. Thus, any surface patch outside the sphere $\mathbf{O}(\mathbf{p}, ||\mathbf{pq}||)$ can be directly eliminated. We also call sphere $\mathbf{O}(\mathbf{p}, ||\mathbf{pq}||)$ the clipping sphere. During the subdivision process, the point **q** becomes more and more close to the test point **p** and the radius of the clipping sphere becomes smaller and smaller, which will lead to higher and higher elimination efficiency.

One of the key issues for the spherical clipping method is to judge whether a surface patch is outside a sphere. If all the control points of a surface patch are inside a sphere, then the patch must be inside the sphere too. Thus, the clipping sphere seems to be useful to compute the maximum distance between a point and a surface. Unfortunately, even if all the control points of a surface are outside a sphere, we can't ensure that the surface is outside the sphere. It seems not easy to judge whether a surface is outside a sphere directly by its control net.

To overcome this problem, we introduce the objective squared distance function for judging whether a surface is outside a sphere. By using the product formula of two B-spline basis functions in [6], the objective square distance function

$$F(u,v) = (\mathbf{p} - \mathbf{S}(u,v))^2$$

can be turned into B-spline form

$$F(u,v) = \sum_{i=0}^{\kappa}\sum_{j=0}^{\iota} \hat{B}_{i,2p}(u)\hat{B}_{j,2q}(v)f_{i,j}, \tag{3}$$

where the new knot vectors $\hat{U}$ and $\hat{V}$ for $\hat{B}_{i,2p}(u)$ and $\hat{B}_{j,2q}(v)$ are