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Graphical Models 68 (2006) 324–332
Graphical Models
www.elsevier.com/locate/gmod

Swirling-sweepers: Constant-volume modeling ☆

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Received 16 March 2005; accepted 30 November 2005
Available online 29 March 2006

Abstract

Swirling-sweepers is a new method for modeling shapes while preserving volume. The artist describes a deformation by dragging a point along a path. The method is independent of the geometric representation of the shape. It preserves volume and avoids self-intersections, both local and global. It is capable of unlimited stretching and the deformation can be constrained to affect only a part of the model. We argue that all of these properties are necessary for interactive modeling if the user is to have the impression that he or she is shaping a real material. Our method is the first to implement all five.

Keywords: Shape deformation; Volume preservation

1. Introduction

In a virtual modeling context, there is no material: no wax, clay, wood or marble. A challenge for computer graphics is to provide a virtual tool that convinces the artist that there is material. To perfect this illusion, the shape must behave in accordance with a suitable modeling metaphor.

Volume is one of the most important factors influencing the manner in which an artist models with real materials. A virtual tool preserving volume is needed to help the artist believe he is interacting with material. Also, modeling by preserving the available amount of material will produce a shape with style, that other virtual modeling methods can only achieve with more effort.

1.1. Previous volume-control models

Volume preservation has been recognized for a long time in animation as a desirable property for the animation of believable animal and human characters [1]. Platt and Barr [2] use constrained optimization methods for objects discretized into lattices. Desbrun and Cani [3] use controllers for maintaining the implicit surface that coats a set of particles to a constant volume during deformation. Foster

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1524-0703/$ - see front matter
doi:10.1016/j.gmod.2005.11.001

and Fedkiw [4] achieve incompressibility in water simulation by maintaining a divergence free velocity field, thanks to the Poisson equation. Teran et al. [5] rely on finite volume methods to simulate quasi-incompressible materials such as muscular tissue.

Volume preservation has also been considered as a very useful constraint for the intuitive modeling of shapes. Rappoport et al. [6] propose an optimization method to adjust the control points of the popular free form deformations (FFDs) [7], but it works only for tensor-solids. Hirota et al. [8] also adjust FFD control points, but their method does not allow local editing. Aubert and Bechmann [9] propose a volume-preserving space deformation based on a model called DOGME. The deformation does not have a local support, and requires the computation of the shape's volume. Botsch and Kobbelt [10] preserve only a volume between the surface and a base surface. Dewaele and Cani [11] introduce mass-preserving local and global deformations for shapes represented by a mass-density field sampled in a grid.

The limitation of existing methods is either that they only apply to a specific type of geometric representation, or they only apply to shapes whose volume can be computed.

1.2. Overview

This paper presents swirling-sweepers, a new method dedicated to modeling shapes while preserving the shape's volume. Our technique belongs to space deformations, and is therefore applicable to a wide range of geometric representations, including all of the popular parametric surfaces.

It is the first method that preserves volume, has a local support, prevents local and global self-intersection of the surface, and does not require any volume computation. Most importantly, using the method is simple: the artist only has to provide the trajectory of a point, for instance with a mouse.

In Section 2, we summarize the principle of the space deformations called sweepers. Then, we present in Section 3 our new method for modeling by constant-volume deformation. Finally, we propose a shape representation suitable for interactive shape modeling in Section 4.

2. Principle of sweepers

We briefly review the elements required for understanding the space deformations called *sweepers* [12].

Space deformation provides a formalism to specify any modeling operation by successively deforming the space in which an initial shape, $S(t_0)$, is embedded. A deformed shape is given by the *modeling equation*[2]:

$$S(t_n) = \left\{ \Omega_{i=0}^{n-1} f_{t_i \mapsto t_{i+1}}(p) | p \in S(t_0) \right\}, \tag{1}$$

where $f_{t_i \mapsto t_{i+1}} : \mathbb{R}^3 \to \mathbb{R}^3$ are n space deformations, deforming a point p of shape $S(t_i)$ into a point of shape $S(t_{i+1})$.

Sweepers provide a convenient formalism to define deformation functions $f_{t_i \mapsto t_{i+1}}$ useful for shape modeling. The user defines a sweeper deformation simply by moving a geometric tool with a gesture.

Informally, a sweeper deformation is a geometric tool together with a motion path. This tool defines an influence function that describes the magnitude of the tool's effect on space. The motion of the tool drags a part of space defined by the influence function, in a manner that prevents the surface of the deformed shape from self-intersecting. This property is relevant: a surface self-intersection is undesirable since it produces an incoherent shape, and also because a space deformation cannot remove a self-intersection.

More formally, a simple sweeper deformation is defined by a scalar function, $\phi_t : \mathbb{R}^3 \mapsto [0,1]$, that varies over time, t. This field can be defined conveniently by composing the distance to the tool d_t, with an influence function μ:

$$\phi_t = \mu \circ d_t. \tag{2}$$

Any smooth decreasing function of finite support can be used for μ. We chose a C^2 continuous piecewise polynomial, in which λ defines the radius of the influence:

$$\mu_\lambda(d_t) = \begin{cases} 0 & \text{if} \quad \lambda \leqslant d_t, \\ 1 + \left(\frac{d_t}{\lambda}\right)^3 \left(\frac{d_t}{\lambda}\left(15 - 6\frac{d_t}{\lambda}\right) - 10\right) & \text{if} \quad \lambda > d_t. \end{cases} \tag{3}$$

The tool's motion is defined by transforming the tool's position, size, and orientation, given by the matrix M_{t_i} into the next configuration, given by the matrix $M_{t_{i+1}}$. Let us denote $M_i = M_{t_{i+1}} \cdot M_{t_i}^{-1}$ the transformation matrix from the previous to the new configuration. A naive deformation of a point p with a single tool would be:

[2] $\Omega_{i=0}^{n-1} f_{t_i \mapsto t_{i+1}}(p)$ expresses the finite repeated composition of functions $f_{t_{n-1} \mapsto t_n} \circ \cdots \circ f_{t_0 \mapsto t_1}(p)$.