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Realistic photometric stereo using partial differential irradiance equation ratios

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ABSTRACT

Shape from shading with multiple light sources is an active research area and a diverse range of approaches have been proposed in the last decades. However, devising a robust reconstruction technique still remains a challenging goal due to several highly non-linear physical factors being involved in the image acquisition process. Recent attempts at tackling the problem via photometric stereo rely on simplified hypotheses in order to make the problem solvable. Light propagation is still commonly assumed to be uniformly oriented, and the *BRDF* assumed to be diffuse, with limited interest for materials giving specular reflection. Taking into account realistic point light sources, in this paper we introduce a well-posed formulation based on partial differential equations for both diffuse and specular shading models. We base our derivation on the popular approach of image ratios, which makes the model independent from photometric invariants. The practical effectiveness of our method is confirmed with a wide range of experiments on both synthetic and real data, where we compare favorably to the state of the art.

1. Introduction

Nowadays, the importance of having three-dimensional objects on a computer directly imported from the real world is increasing due to the fact that many applicative fields need to observe, manipulate or reproduce reality. However, among the several existing techniques for 3D scanning, only few can be actually considered as emerging technologies in the market. Structured light [1] and multi-view stereo [2] are gradually moving to smart devices, making them able to provide a rough depiction of a depth field rather than an accurate 3D reconstruction. Reconstruction accuracy depends on a number of factors which may prevent a precise measurement of the observed scene, and for these reasons additional constraints need to be imposed on the acquisition environment.

Photometric stereo (PS) allows highly accurate reconstructions under the requirement to work in a controlled setup, making it applicable in limited scenarios. However, starting from the seminal work of Woodham [3], the use of complex shading models has not evolved much in modern PS approaches [4,5]. Indeed, the simplifying approach that models the captured images as an inner product between uniform light direction and outgoing surface normal is still commonly found in the literature [6]. Shape reconstruction from shading information is a difficult problem, due to the complexity of the underlying physical process describing how a light beam bounces on the surface. Thus, it becomes fundamental to take into account the parametrization of all elements that influence the image formation. Although the behavior of light itself (e.g., propagation and attenuation) does require to be carefully modeled, the bidirectional reflectance direction function (BRDF) represents the true bridge between the real world and the depicted one. Most of the literature dealing with PS assume diffuse reflection (i.e., uniform in all directions), reducing the mathematical model to a linear problem where the normal field can be easily computed [7] and finally integrated [8]. Realistically, this approach contains too many assumptions which fail as soon as the method is applied in many real-world applications. There are at least two reasons why the reconstruction of *specular* surfaces still remains a challenging task in the PS field. First, the BRDF for specular reflections is highly non-linear, which means that mathematically it is difficult to have a straightforward solvability. Second, the specular reflection has a very susceptible signal (see Fig. 1, upper part), easily corruptible by environment and acquisition noise.

Instead of considering shape reconstruction in the wild [9,10], in this paper we take into account a controlled lighting setup in which point light sources are used to illuminate the observed object. This kind of setup is commonly used in the PS field [11,12] by virtue of its applicability in several interesting applications [13–15]. We introduce a new differential model which allows to extract

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Fig. 1. Full reconstruction of the “bimba” model from 6 partial views under 5 point light sources. The images used for the reconstruction (top, one light source only) consist solely of the *specular* component of the reflected light, yet our method provides a near-perfect reconstruction of the model. The artifact on top of the head is due to the surface region not being visible in any of the partial views. See Section 4.1 for more details on the reconstruction process.

shape information even for specular surfaces (Fig. 1). The approach is based on the Blinn–Phong shading model [16], which has been shown to be a realistic BRDF [17]. The model we propose improves upon a recent proposal [18], which only takes into account *diffuse* surfaces. Similarly to this previous work, we formulate the problem using non-linear PDEs as our main mathematical tools.

1.1. Related work

As mentioned in the previous section, dealing with general BRDFs is a challenging problem due to the non-linearities arising in the image formation process. Previous studies consider an object of the same material as a reference [19], or attempt to get rid of the specularity by eliminating highlights in a pre-processing step [20,21]. Dichromatic reflectance models have also been considered as a basis for diffuse and specular reflection separation, in particular [22] makes use of a dark channel prior and [23] iteratively compares the intensity logarithmic differentiation of the input image. The dichromatic model can be employed to perform shape reconstruction of the diffuse component only [24,25]. Yang and Abuja [26] use both the diffuse and specular components for the reconstruction by assuming the illumination chromaticity to be known, and at least one of the input images to be free of specularity.

Several works take into account general irradiance equations, leading to difficult problems that are then solved after substantial simplifications. For example, Ikehata et al. [4,5] use the purely diffuse irradiance equation for general surfaces, and consider the specular component as a sparse error. In order to provide an accurate solution, they introduce a regression procedure which requires tens of images. The use of such a big amount of data partially justifies the assumption of ignoring specular reflection (due to its sparsity). On the other hand, expensive algorithms based on energy minimization tend to be slow and cannot provide real-time shape recovery. Furthermore, light propagation is considered to be uniform, not allowing realistic features such as radial propagation and attenuation of light to be taken into account.

When the linearization of real physical effects comes into play, it induces deformations on the recovered surface. Motivated by this, we formulate a new PS differential model for specular surfaces that takes into account point light sources. For this purpose we make use of non-linear PDEs, a mathematical tool that has attracted increasing interest in the last few years [27,28,11]. These approaches consider *image ratios* in order to yield photometric invariants, and model the irradiance equations via PDEs. Mecca et al. [27,28,18] use specific irradiance equations for diffuse surfaces, and the uniqueness of solution is proved by characteristic strip expansion and assuming known light information (*i.e.*, direction or position). Chandracker et al. [11,29] consider more general irradiance equations with unknown light sources, and compute the photometric invariants describing the surface through its isocontours [30]. However, the shape reconstruction process requires additional initial or boundary information.

All the works mentioned so far take into account *uniform* light propagation, which restricts the problem to a very specific and controlled environment. Parametrization of realistic point light sources is not new [15,31], yet very few approaches apply this idea to shape reconstruction. Wu et al. [13] use two point light sources