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Algorithm for computing positive α-hull for a set of planar closed curves

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ABSTRACT

In this paper, the computation of positive α-hull for a set of planar closed C^1-continuous curves has been addressed without sampling the curves into point-sets or polylines. Positive α-hull, so far, has been computed only for a set of points, using the farthest Delaunay triangulation, a dual of farthest Voronoi diagram. However, Delaunay triangulation does not exist for a set of curved boundaries and the computation of Voronoi diagram for such a set is still a topic of active research. The key insight behind our algorithm is to merge adjacent pairs of curves on the convex hull into a set of triplets. Along with a directed-cyclic graph and a R-List (list of radii), α-neighbours are derived. Using the constraint equations, α-discs are then computed. The algorithm is first provided for convex non-intersecting closed curves, but later explained how it can be generalized for non-convex curves. We show that the algorithm has time complexity of $O(n^2)$ time where n is the number of curves, which leads to a practical implementation with a reasonable running time in seconds for a few dozen curves. By directly operating on the curves, our method is both robust and accurate thus avoiding the problems that arise on polyline/point-set approximations of the curve networks.

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1. Introduction

α-hull [1] for a point-set is a shape that generalizes the notion of convex hull (in this paper, the term 'convex hull' is used synonymous with the boundary of the convex hull [2]). The α-hull uses a real parameter α, variations of which leads to a family of shapes. α-hull's discrete counterpart is α-shape [3], a popular characterisation for a set of points.

Planar curves represented by NURBS have been useful in applications such as profile cutting or contour milling [4] and in high speed machining. Such applications often require computations like collision detection for tool path planning. Bounding hulls, which typically bound the input set, such as convex hull, positive α-hull, enclosing entities viz. rectangles circles/spheres, ellipses, etc. have been found useful in such computation. This is because, these hulls are either composed of straight lines or circles, which are easier to be employed for computational purpose. In particular, enclosing circles/discs enable the speed up of the computations such as collision detection as the check can be reduced to a computing distance to the center of the discs/circles.

Computations of bounding hulls often belong to computational geometry have been typically considered on a set of points or polygons as input. However, they have now been extended to other fields such as CAD, geometric modeling with domains like closed curves and surfaces (e.g. medial axis [5], minimum spanning hypersphere [6], etc.). It is imperative to note that inputs such as curves also call for different kind of approach for the same problem for a set of points. For example, traditional approach to solve medial axis for a set of points such as bisectors could prove expensive for curves. Hence, other approaches such as tracing [5] have to be employed. Computations such as convex hull [7], visibility graph [8] and shortest path [9] require tangents and bi-tangents for a set of curves, which is not the case for a set of points. The computations are typically much more numerically intensive than its point-set counterpart. For example, convex hull computation [7] has been formulated as finding zero-sets of polynomial equations in two variables (for bi-tangent identification). Since all the computed bi-tangents for a curve may not be part of the final convex hull, steps for eliminating redundant bi-tangents have also been discussed.

Traditionally, construction of α-hull/shape [1] for a set of points has been based on Delaunay triangulation. It should be noted that, though the theoretical foundations for Voronoi diagram of a set of freeform curves have been well-developed in the recent past, computing Voronoi diagram for such inputs is still a topic of active research (for example, refer to [10] for Voronoi cell). Moreover, Delaunay triangulation for a general set of curves does not exist and it is possible only if the curves are discretized into a set of

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points. Even the generalized Voronoi diagrams can at most deal with circular arcs [11] and not applicable for any generic set of curves.

One approach to find the α-hull for a set of curves is to generate set of points on the curves and then use an existing algorithm. However, this approach may result in a very coarsely approximated hull (depending on the sampling on each curve) of the input curves which might impede the accuracy of the results (this argument has been shown to be true for algorithms such as medial axis [5]). Hence, the following are the motivations for this work:

- No algorithm seems to exist for computing α-hull for a set of curves that are represented exactly (i.e. without approximating using sample points).
- Algorithm for computing farthest Voronoi diagram for curves has not yet been developed so far.
- Voronoi diagram, in the curved domain, is still quite difficult to compute accurately.
- Delaunay triangulation for a set of curves does not exist as the Voronoi diagram may not consist of just straight lines.
- Approximate representation (such as sampling into points or polylines) of the curves will lead to inaccuracies in the computation.

In this paper, an algorithm has been developed for computing positive α-hull of a set of curves without employing Voronoi/Delaunay based computation. The curves used are represented using non-uniform rational B-spline (NURBS). It is to be noted that the closed curves have well defined exterior and interior unlike that of points. It is assumed that the interior of a closed curve lies to its left as we travel along the increasing direction of parametrization.

2. Preliminaries

Initially, the definitions are given for a set of points for ease of understanding and fine-tuned for a set of curves. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of points termed as *sites*. The α-hull for the set P can be defined in the following manner [1].

Definition 1. Let α be a sufficiently small but otherwise arbitrary positive real. The positive α-hull of P is the intersection of all closed discs with radius $1/\alpha$ that contain all sites of P.

Definition 2. For arbitrary negative reals, the negative α-hull is defined as the intersection of all closed complements of discs (where these discs have radii $-1/\alpha$) that contain all the sites.

Essentially, the positive α-hull is formed out of discs, all of them contain all the sites (Fig. 1(a)). On the other hand, negative α-hull for a set of sites are formed by discs, where any closed disc does not contain any site (Fig. 1(b)).

A note on the notation used: It is to be noted that Definitions 1 and 2 were based on [1], where the reciprocal notation for the radius (i.e. $1/\alpha$) has been used. Both negative and positive α-hulls use closed discs, whose radius value is typically a positive one (the negative notion in Definition 2 is more of a symbolic one). Positive and negative α-hull depend on whether a closed disc contains the set or not respectively. In subsequent works (such as [3]), researchers have used negative α-hull synonymous with α-hull (in fact, α-shape, α-hull's discrete variety [1,3]) without the reciprocal notation and hence the distinction between positive and negative α-hulls has not been given that much emphasis. Nevertheless, in this paper, hereafter, the notation α is used to denote the radius of the closed disc, a positive value (dropping the reciprocal notation) and also for computing the positive α-hull (unless mentioned otherwise).

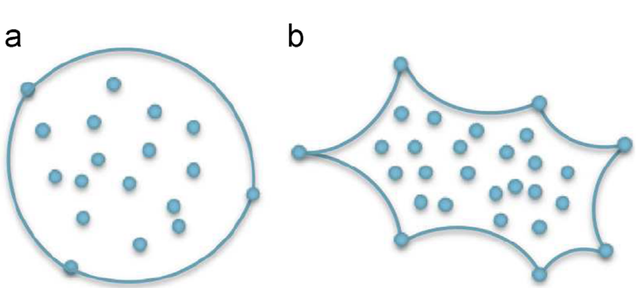

Fig. 1. α-hulls for a set of points [1]. (a) Positive α-hull. (b) Negative α-hull.

2.1. Set of curves as input

Let S be a set of free-form (parametric) curves with no straight line portions and having no discontinuities in R^2. The positive α-hull of S can be defined in the following manner.

Definition 3. α-disc is a disc of radius α.

Definition 4. Let α be a positive real so that there exists a disc of radius α that contains all input curves. The positive α-hull of S is the intersection of all such discs.

Definition 5. Two curves in the set that touch α-disc contiguously are called α-neighbours.

Definition 6. An enclosing disc is a disc that encloses (contains) the entire set of given curves.

Definition 7. A disc that encloses three input curves (termed as *triplet*) is called a triplet disc for these curves (i.e., the disc may or may not enclose all the other curves in the set).

Definition 8. The minimum enclosing disc (MED) of a set is the smallest radius disc that encloses the entire set.

In this paper, the radius of MED is denoted as RMED. It is well known that, in the case of a set of points, farthest Voronoi diagram and farthest Delaunay triangulation play a role in the computation of positive α-hull with the result that only points on the convex hull need to be used for computation of positive α-hull [1].

Definition 4 indicates that, for a given set of curves S, α-disc contains all the curves in the set. In the case of a set of sites, the boundary of the α-hull occurs at all the places where the α-disc stays in contact with at least two different sites. However, this is not true for a set of curves, in general, i.e. the α-disc may touch the same curve at more than one point for positive α-hull. To account for this, the curves are split at local minimum positive curvature points as well as at inflection points (for non-convex curves). The splitting of curves ensure that the α-disc touch each split portion at only one point. This enables the algorithm to handle complex shapes typically used in CAGD domain.

Though strong relation exists between farthest Voronoi diagram (fVD) [2] and positive α-hull, it is to be noted that no known algorithm exists for computing farthest Voronoi diagram accurately for freeform curves. Delaunay triangulation (neither closest nor farthest) for a set of curves is not defined, making the identification of α-neighbours not that straight forward for such a set.

To the best of the knowledge of the authors, no known algorithm exists for the computation of positive α-hull for a set of curves (from now on, the input set is a set of curves, unless otherwise mentioned). In this paper, the focus is on getting α-neighbours from a set of closed curves without computing the farthest Voronoi diagram of curves. Constraint equations to identify where α-disc touches for pairs, triplets etc., adopting from [6] have also been discussed briefly.

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