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Scale-space feature extraction on digital surfaces ☆

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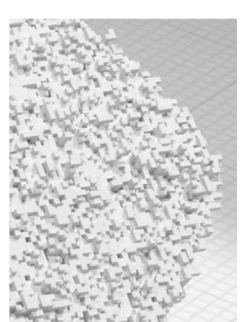
ABSTRACT

A classical problem in many computer graphics applications consists in extracting significant zones or points on an object surface, like loci of tangent discontinuity (*edges*), maxima or minima of curvatures, inflection points, etc. These places have specific local geometrical properties and often called generically *features*. An important problem is related to the scale, or range of scales, for which a feature is relevant. We propose a new robust method to detect features on digital data (surface of objects in $\mathbb{Z}^3$, which exploits asymptotic properties of recent digital curvature estimators. In Coeurjolly et al. [1] and Levallois et al. [1,2], authors have proposed curvature estimators (mean, principal and Gaussian) on 2D and 3D digitized shapes and have demonstrated their multigrid convergence (for C^3-smooth surfaces). Since such approaches integrate local information within a ball around points of interest, the radius is a crucial parameter. In this paper, we consider the radius as a scale-space parameter. By analyzing the behavior of such curvature estimators as the ball radius tends to zero, we propose a tool to efficiently characterize and extract several relevant features (edges, smooth and flat parts) on digital surfaces.

1. Introduction

When performing geometry processing on shapes, a classical problem in many computer graphic applications consists in delineating places with specific local geometrical information—or *features*—on the shape surface. Even if no clear definition of feature on surface stands out, prior works usually characterize a feature as a local discontinuity distinguishable from its neighborhood. As an example, differential quantities have been widely considered in this context as preliminary information from which features can be extracted. However, an important problem is related to the scale (or range of scales) for which a feature is relevant. This question leads to scale-space analysis of shapes. Note that this concept has been widely investigated in the image processing community [3].

In this paper, we propose a new robust feature extraction technique which incorporates scale-space geometrical information and which is dedicated to digital surfaces (boundary of objects in $\mathbb{Z}^2$ or $\mathbb{Z}^3$). We consider raw digital data (shapes discretized on a regular grid) as an input for two main reasons: First, many acquisition devices (*e.g.* 3D MRI images or X-ray tomography) provide such data and we do not want to introduce approximations or interpolations by switching to a polyhedral representation. Second, working on digital data allows us to consider a mathematical framework—the multigrid convergence of operators—dedicated to this digital model. Specific in the sense that boundaries of volumetric objects usually lead to a large number of surface elements. Furthermore, due to the digitization effect, digital surfaces can be considered as approximations of continuous manifolds with a very specific and isothetic noise model: samples are evenly spaced but never lie on the surface, normals are not informative. This kind of data could be problematic if it is not carefully handled when defining differential estimators for instance. Finally, this case study is motivated by accurate shape analysis of 3D volumetric porous material (microstructures of snow samples, see Fig. 7). In this context, we want to characterize geometrical discontinuities (edges) from smooth areas and zero curvature (flat) regions in a robust way.



Related works: First of all, shape discontinuities can be formalized as *ridges and valleys* with differential geometry. In this case, such discontinuities are deduced from differential quantities of order 3 by looking at variations of principal curvature directions in a neighborhood [4,5]. The final step consists in thresholding significant angular deviation of principal directions. Such techniques provide a formal approach to disconti nuities extraction but are scaledependent and rely on a robust

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estimation of order 3 differentials. When dealing with noisy data or digital data, such approaches are not relevant and cannot be considered.

For meshes or point clouds, many approaches are based on integral quantities computed on local patches. For instance, Pauly et al. [6] and Clarenz et al. [7] have used Principal Component Analysis on data points located in a given neighborhood of the point of interest. A feature score is defined as a function of the eigenvalues of this covariance matrix. Then, either the feature score is simply thresholded, or the behavior of the score as a function of the neighborhood size is analyzed. Mérigot et al. [8] extended this approach to consider convolved covariance matrices of Voronoi cells (*Voronoi Covariance Measure* or VCM). Thresholding a ratio of VCM eigenvalues leads to a robust extraction of edges on point clouds or meshes. Such approaches produce interesting results at a fixed scale or for smooth objects. However, scale-space analysis is not fully integrated in these frameworks. Furthermore, even if ratios of covariance matrix eigenvalues are related to principal curvatures, the geometrical interpretation of scores is not straightforward. In the experimental section, we provide more details on this approach.

In a similar way, Park et al. [9] have proposed a Tensor-Voting strategy on local surface patches. They used the scale-space behavior of the tensor vote when the neighborhood size increases, in order to extract edges on point clouds. As shown in the experiments, this technique is very sensitive and does not provide sufficiently robust results on digital surfaces. Mellado et al. [10] have introduced a fast least square spherical fitting approach to a point cloud to create a multi-scale feature score. Again, the scale-space parameter is the neighborhood size considered in the fitting. Even if this feature score is qualitatively relevant, it is not directly related to some geometrical information. Furthermore, when used on digital data, such technique fails to provide a precise localization of features.

Finally, features can be extracted following a spectral analysis of the shape from eigenvalues of the surface Laplacian matrix [11–13]. In this context, features are characterized by spectral quantities which are locally stable and distinguishable from its neighborhood. Such techniques are very promising but drawbacks exist for digital surfaces. First, since our surfaces have a large number of elements, computing the eigenvalues of the Laplacian matrix could be very computationally expensive. Another bottleneck relies on the fact that for digital surfaces, the isothetic nature of the Euclidean embedding (digitization on axis aligned grid) makes the metric not well embedded in the discrete Laplacian operator. Indeed, if we consider the DEC formulation or simply the *cotan* approach to define a discrete Laplacian operator on the digital surface embeddings, the staircase effect of the digitization makes the metric not well described by the geometrical embedding of the surface. For example, a consequence is that heat diffusion obtained by this operator produces anisotropic artifacts (ellipsoidal isocontours on a digital plane with normal vector $(1,1,0)^T$ for instance). On digital surfaces, a discrete Laplacian operator with correct intrinsic metric information has to be defined.

Contributions: We propose a robust scale-space feature selector that classifies digital surface elements into three categories: *edge*, *smooth* or *flat*. This feature selector is built upon digital curvature estimators and relies on their theoretical multigrid convergence properties. Since these estimators are parametrized by the size of their ball of integration, *i.e.* a kind of scale, the feature selector analyses curvature estimated as a function of scales. Since we know the theoretical behavior of models *edge*, *smooth* and *flat*, the feature selector chooses the model that best fits its input data. We compare our approach on a large class of shapes with the other above-mentioned approaches to feature detection, and we evaluate their robustness to noise. Finally, we apply this feature selector to the analysis of microstructures of 3D snow samples.

2. Preliminaries

In Geometry Processing, *integral invariants* have been widely investigated to construct estimators of differential quantities on smooth surface [14,15]. The main idea is to move a ball B_R of radius R on points x of the boundary ∂X of shape X. Then, integrals are computed on the intersection between this ball and the shape, *i.e.* on $B_R(x) \cap X$ (see Fig. 1a for notations). More formally, by Taylor expansion of the area and volume around the point x, 2D curvature estimator $\tilde{\kappa}_R(x)$ and 3D mean curvature estimator $\tilde{H}_R(x)$ can be defined respectively as [14]

$$\tilde{\kappa}_R(X,x) \overset{def}{=} \frac{3\pi}{2R} - \frac{3A_R(x)}{R^3}, \quad \tilde{H}_R(X,x) \overset{def}{=} \frac{8}{3R} - \frac{4V_R(x)}{\pi R^4}, \tag{1}$$

where $X \subset \mathbb{R}^2$ (resp. $\mathbb{R}^3$) is a sufficiently smooth shape. Here $A_R(x)$ is the area and $V_R(x)$ the volume of $B_R(x) \cap X$ (*i.e.* we integrate the unit constant function on $B_R(x) \cap X$). $\tilde{\kappa}_R(X,x)$ and $\tilde{H}_R(X,x)$ values converge to expected ones (respectively curvature κ and mean curvature H) as R tends to zero [14], since

$$\tilde{\kappa}_R(X,x) = \kappa(X,x) + O(R), \quad \tilde{H}_R(X,x) = H(X,x) + O(R). \tag{2}$$

Similarly, principal curvatures can be estimated by computing the two greatest eigenvalues λ_1 and λ_2 of the covariance matrix of $B_R(x) \cap X$ [14]

$$\tilde{\kappa}^1(X,x) \overset{def}{=} \frac{6(\lambda_2 - 3\lambda_1)}{\pi R^6} + \frac{8}{5R} + O(R), \tag{3}$$

$$\tilde{\kappa}^2(X,x) \overset{def}{=} \frac{6(\lambda_1 - 3\lambda_2)}{\pi R^6} + \frac{8}{5R} + O(R). \tag{4}$$

Using similar integration principles, several estimators of various differential quantities can be defined. Refer to [15,16] for an overview.

2.1. Integral based digital curvature estimators

In our context, we consider digital shapes (any subset of $\mathbb{Z}^d$) and boundaries of digital shapes. We denote by $\mathrm{D}_h(X)$ the Gauss digitization of X in a d-dimensional grid with grid step h, *i.e.* $\mathrm{D}_h(X) = X \cap (h\mathbb{Z})^d$. For such digitized set Z, $Bd(Z)$ denotes its topological boundary, seen as a cellular Cartesian complex (see Fig. 1b). It is thus composed of 0-cells and 1-cells (resp. *pointels* and *linels*), and, for $d=3$, with 2-cells (*surfels*), embedded in the digital grid.

Before going further, we define the 2D digital curvature estimator $\hat{\kappa}_R$, the 3D digital mean curvature estimator $\hat{H}_R$ and the 3D digital principal curvature estimators κ_R^1 and κ_R^2 on $Z \subset \mathbb{Z}^2$ or $Z \subset \mathbb{Z}^3$.

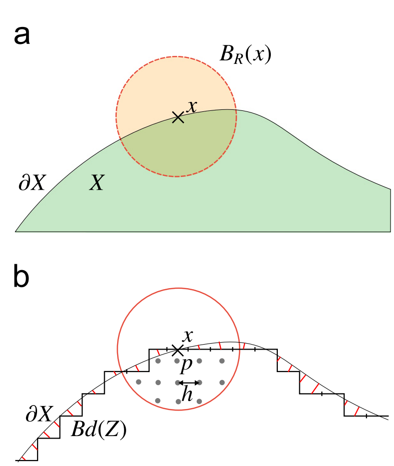

Fig. 1. Integral in variant computation (a) and notations (b) in dimension 2 [1].