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Shape conforming volumetric interpolation with interior distances

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ABSTRACT

Source based heterogeneous modelling is a powerful way of defining gradient materials within a volume. The current solutions do not take into account the topology of the object and can provide counter intuitive results for complex objects. This paper presents a method to interpolate material properties and attributes based on the accessibility of the points with respect to the material features defined by the user. Our method requires the nonoverlapping source features with constant material to interpolate gradient materials, by using Voronoi diagrams on interior distances. It leads to intuitive material properties across the shape regardless of its topology or complexity. We show how the shape conforming field is defined inside the volume and can be extended outside the volume to create a valid operator for a heterogeneous modelling system dealing with scalar fields. The presented method is computationally efficient and has several applications, such as material property interpolation and shape aware procedural micro structures.

1. Introduction

Recent developments in the representation of real-life objects show a paradigm shift from surface modelling where the interior of the object is homogeneous to a heterogeneous volume modelling approach. This allows us to take into account the complex internal structure of the object which can contain diverse materials and possess various physical properties. Heterogeneous modelling has found its applications in a wide range of diverse areas from biological and medical research and bio-engineering to multi-material design for fabrication and digital entertainment applications.

There exist several ways to define the interior structure of the object. The simplest one is explicit definition, e.g. for a large number of points that belong to the interior of the object we can specify the values of the attributes such as material and density. Alternatively if the geometry of the object is represented by a voxel set, the attributes can be defined per voxel. However in practice explicit definition is very inefficient because the more precise the materials are defined the more points or voxels should be set and the amount of data can be unbearable in this case. Therefore procedural methods becomes are more suitable for the purposes of the material definition. A convenient way to do this is to specify the attributes only in a finite number of points or closed areas and interpolate the values elsewhere. Most of the interpolation methods, such as transfinite interpolation [1] however, do not take into account the shape of the object itself but rather providing a general interpolation within the whole space.

In this work we present a method that allows to define the interior structure of the object by using simple interpolation techniques yet taking the shape of the object into an account. We are working with the object defined by continuous scalar fields that allows us to work with arbitrary accuracy without increasing the complexity. As a result, we allow the user to set up the materials in a convenient way and efficiently evaluate the material in arbitrary point in space inside as well as outside the volume object. In this paper we present formulations for shape-aware material distribution for the volume object represented by a scalar field, discuss the practical approach for interior distance fields and present some applications of our method for heterogeneous modelling.

2. Background and related work

2.1. Procedural material distribution

Different approaches exist to define the material distribution inside a volume object using limited information. Some methods use the attributes at the surface to extrapolate the attributes inside the volume by propagation. Thus, in [2] the texture coordinates for the point inside a source shape are obtained by finding the corresponding point on the support surface by letting particles flow through the gradient vector field. In general, this method can

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lead to discontinuities and is very limited in modifying attributes inside the volume. Source-based methods allow users to specify more information regarding attributes inside the shape by defining the sources for the material distribution and procedures to find the values of the attributes in the ambiguous areas where no source has been set up. Heterogeneous modelling using source features was introduced in [3] where the standard set-theoretic operations were used in the context of material properties. The gaps in the material distribution were filled by using a weighted sum of the materials defined by the neighbouring source features. In [4] the material distribution is defined by using optimisation techniques with parametric distances. In [5] is shown how the particular attributes of the model can be interpolated by using mass transport. However the generalisation for arbitrary attributes is not straighforward and the solution requires solving PDE systems for querying the attribute value in a point in space which makes it impractical for interactive applications. In [1] transfinite interpolation is used to define the material properties in space including the interior of the object. The simple formulation is used to find the attributes in the areas between the sources, however the method is global and does not take the shape of the object into account. The shape of the object can was taken into an account for a limited types of input data. Thus, in [6] the medial axis is used to define shape-aware internal structure, which makes it uneasy to use for the geometry where the medial axis is hard to find, e.g. implicitly defined.

2.2. Interior distance fields

Interior (inner) distance measures the path between two points inside the object where all the points of the path belong to the interior or the surface of the object. Various ways to define and calculate the inner distance between two points belonging to a closed object exist, including barycentric coordinates of a polygonal mesh [7] and a visibility graph for the volumetric object [8]. Also, the inner distance can be defined by using other ways, for example, in terms of Eikonal equation [9] and heat flow [10]. In general, as noted in [7], the interior distance can be expressed in a continuous setting, however in practical applications usually approximations are used.

3. Shape-aware interpolation

In our method we see the material distribution as a function $f_m : \Re^3 \mapsto \Re^n$ where n denotes the number of real numbers to describe the attributes in the material distribution. In the simplest case the material is defined by a real-valued function, i.e. $n=1$, the examples of this material can be value of the material density or the contrast of the colour defining the material (K component). In this paper for visualisation purposes for most of the examples we define the material by its RGB colour, $n=3$.

We are working with the geometry defined in an implicit form, that means the volume object can be represented by its boundary (e.g. polygonal mesh), but we convert this boundary to a function for which the sign of the value in the given point denotes whether the point in question lies inside, outside or on the surface of the volume. Given the volume object S, the shape of the volume object is defined as a scalar field with a defining function $f_s(\mathbf{x})$. The sign of the defining function distinguishes the interior and the exterior of the volume, i.e. $f_s(\mathbf{x}) > 0$ for the exterior, $f_s(\mathbf{x}) < 0$ for the interior and $f_s(\mathbf{x}) = 0$ for the boundary of the volume object.

Inside the volumetric object, we define a number of material features, i.e. closed regions where the geometry and the material are known. The geometry of each material feature is also represented as a scalar field with the defining function $f_{m_i}(\mathbf{x})$, where $i = 1 \dots n$ is the index of the given material feature and n features are defined overall (Fig. 1).

The steps of the method are shown in Fig. 2. In brief, to make the interpolation, we use material source features (Fig. 2a) to propagate the interior distance field with respect to the input shape. Next, we construct a Voronoi diagram based on these distance fields (Fig. 2b) to build attribute cells for each material source and interpolation zones (Fig. 2c). Finally we use an interpolation between the scalar fields of the resulting Voronoi cells (Fig. 2d). Below we describe our method in detail.

3.1. Interior distance field for material features

In our work we use an approximate Euclidean shortest path [11] to define the interior distance field. The choice of this method over other existing in current state of the art (see Section 2.2) is that it is easiest in terms of implementation and as efficient as some others (e.g. [9]).

Given the material feature i with the defining function $f_{m_i}(\mathbf{x})$, the value of the field for this material feature is defined as follows:

$$d_i(\mathbf{x}) = \min_{\forall \mathbf{x}_j : f_{m_i}(\mathbf{x}_j) = 0} \mathcal{L}(\rho(\mathbf{x}, \mathbf{x_j})) \quad (1)$$

Here $\rho(\mathbf{x}, \mathbf{x_j})$ denotes Euclidean shortest path between points $\mathbf{x}$ and $\mathbf{x_j}$ inside the volume object S and $\mathcal{L}$ denotes length of this path.

In practice the exact calculation of Euclidean shortest path is an NP-hard task and given the additional complexity of the global

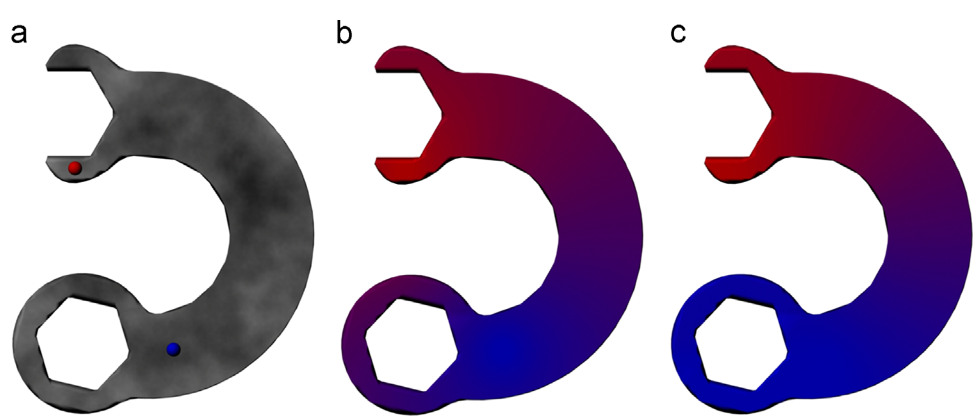

Fig. 1. Material interpolation: (a) Model with material features denoted, (b) global transfinite interpolation, and (c) our method.