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Properly constrained orthonormal functional maps for intrinsic symmetries

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ABSTRACT

Intrinsic symmetry detection, phrased as finding intrinsic self-isometries, courts much attention in recent years. However, extracting dense global symmetry from the shape undergoing moderate non-isometric deformations is still a challenge to the state-of-the-art methods. To tackle this problem, we develop an automatic and robust global intrinsic symmetry detector based on functional maps. The main challenges of applying functional maps lie in how to amend the previous numerical solution scheme and construct reliable and enough constraints. We address the first challenge by formulating the symmetry detection problem as an objective function with descriptor, regional and orthogonality constraints and solving it directly. Compared with refining the functional map by a post-processing, our approach does not break existing constraints and generates more confident results without sacrificing efficiency. To conquer the second challenge, we extract a sparse and stable symmetry-invariant point set from shape extremities and establish symmetry electors based on the transformation, which is constrained by the symmetric point pairs from the set. These electors further cast votes on candidate point pairs to extract more symmetric point pairs. The final functional map is generated with regional constraints constructed from the above point pairs. Experimental results on TOSCA and SCAPE Benchmarks show that our method is superior to the state-of-the-art methods.

1. Introduction

Symmetry is a universal phenomenon in nature which provides global information about the structure of objects. Numerous geometry processing tasks, such as shape matching [1], segmentation [2], geometry completion [3] and meshing [4], benefit from symmetry information. Hence a great deal of work [5] devotes to extract symmetries from geometric data, e.g., point clouds data and polygon meshes.

Most of the previous work concentrates on extrinsic symmetries [6,7]. Recently, intrinsic symmetry detection, phrased as finding intrinsic self-isometries, has received more attention, since intrinsic symmetric objects or phenomenons are more common in real world, such as a human in different poses. However, it is infeasible to search the space of non-rigid transformations directly in classical point-to-point representation. So many methods limit the search space to a set of feature points, and adopt combinatorial algorithms to prune point pairs without preserving local geometric similarity and distance structure [8], which are computationally expensive and sensitive to geodesic noises. Kim et al. [9] take advantage of the fact that intrinsic self-isometries are contained in a low dimensional Möbius transformation space [10] to select the best self-isometry. The symmetry-invariant set, used to generate candidate Möbius transformations, consists of some local extrema of the Average Geodesic Distance function (AGD) [11]. The set may be not perfectly symmetric and leads to failure results. Ovsjanikov et al. [12] extract intrinsic symmetries using functional maps [13]. But they need at least one reference shape with a known symmetry to estimate the quotient space and a consistent decomposition to obtain the final dense intrinsic symmetries. The decomposition divides the shape into fundamental domains, e.g., the right part and the left part of the shape in the case of reflectional symmetry. Furthermore, shapes undergoing considerable degree of non-isometric deformations,

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such as humanoid models with connections between torso and other parts, also challenge the existing methods.

We observe that most existing methods detect intrinsic symmetry over a sparse set of feature points, then propagate the sparse correspondence to the entire shape using geodesic distance. The performance is degenerated since the propagation only considers metric. The functional map framework presents a compact representation of correspondences between shapes, and provides an efficient way to convert functional maps into dense point-to-point correspondences [13]. This motivates us to present an automatic and robust method for global intrinsic symmetry detection leveraging the functional map representation (Fig. 1). Intrinsic symmetries are non-trivial self-isometries represented by orthonormal functional map matrixes. Extending the functional map to detect global intrinsic symmetry directly suffers from the absence of constraints indicating the underlying non-trivial self-isometry. Existing descriptors, such as Heat Kernel Signature (HKS) [14] and Wave Kernel Signature (WKS) [15], provide no valuable cues for distinguishing identity transformation with other symmetry transformations, since they remain invariant in these transformations. Point or segment correspondences contain useful information for distinguishing the above transformations, however the establishment of reliable and enough symmetric point or segment pairs itself is a challenge problem. The key idea of our method is to construct reliable and sufficient regional constraints from symmetric point pairs. The most prominent and stable feature pairs tend to lie on the extremities of the model. We design an initialization procedure to extract sparse and reliable symmetric point pairs from the extremities, and a voting procedure to extract more symmetric point pairs.

In the initialization procedure, initial symmetric point pairs are chosen from a symmetry-invariant set (Fig. 2(a)), which is extracted from shape extremities and whose stability and sparseness make the procedure reliable and efficient. Then we compute an initial functional map satisfying regional constraints, constructed from the initial point pairs. We specify the parts containing the initial symmetric pairs as the reliable parts of the initial functional map. More symmetric point pairs over the reliable parts are selected as symmetry electors (Fig. 2(b)). In the following procedure, a voting scheme is proposed to extract more symmetric point pairs outside the reliable parts (Fig. 2(c)). The final functional map is generated with the regional constraints constructed from all of the point pairs, and converted to a point-to-point mapping (Fig. 2(d)).

When solving for the functional maps, we formulate the problem as an optimization problem with descriptor, regional and orthogonality constraints simultaneously. Compared with refining the functional map by a post-processing [12,13,16], our method does not break other constraints and generates more confident results without sacrificing efficiency. The functional representation, efficient optimization method and effective regional constraints together make our method a faster, automatic

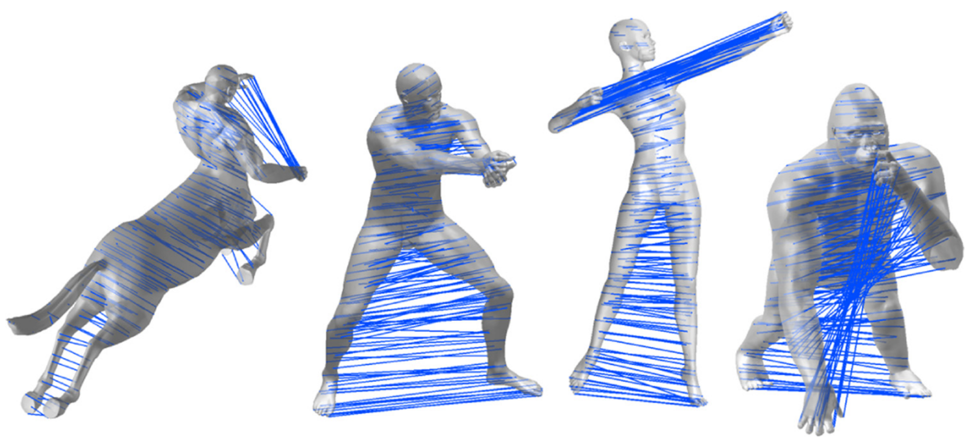

Fig. 1. The results of our method for nearly self-isometric shapes (centaur, michael, victoria and gorilla).

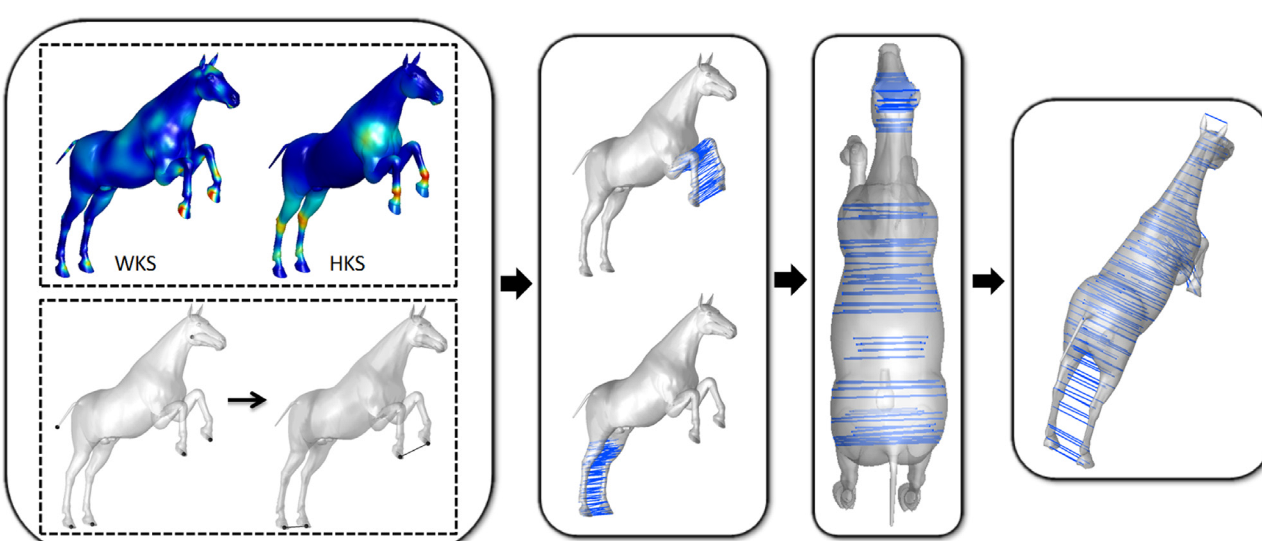

Fig. 2. The pipeline of our method. (a) Initial linear constraints, (b) selection of electors, (c) electors voting and (d) converted to dense self-isometry.