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# Fast as-isometric-as-possible shape interpolation

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ABSTRACT

Shape interpolation, as a bridge communicating static geometries and dynamic shape sequences, is a fundamental operation in digital geometry processing and computer animation. We propose a fast as-isometric-as-possible (AIAP) 3D mesh interpolation approach which casts the shape interpolation problem to finding an AIAP motion trajectory from the start shape to the end shape. This leads to a nonlinear optimization problem with all intermediate shapes as unknowns. The block-coordinate descent method is then employed to iteratively solve the optimization. In each iteration, we need to solve two linear equations whose dimensionality can further be reduced based on a decoupling strategy. Connection maps between orthogonal frames of adjacent edges are further introduced for producing an initial shape sequence in order to address the large-scale deformation problem. A propagation–optimization strategy is then presented to quickly reconstruct the orthogonal frames of all edges from connection maps as well as the orthogonal frame of a specified edge. Refinement of edge quality is available in our method due to the AIAP iterative procedure. In the end, a shape manipulation framework is established for shape sequence transfer and shape sequence editing.



## 1. Introduction

Shape interpolation is to create a sequence of intermediate shapes from two or multiple given poses of an object. The generated sequence should be able to realistically imitate the object's physical motion and meet users' intuitive sensation. Shape interpolation is widely used in computer animation that allows generating a shape sequence from key frames designed by artists. A shape sequence captured via devices such as 3D scanners can be concisely represented by several key frames from which the sequence can then be reconstructed by shape interpolation approaches. Shape interpolation is also a kernel to support other operations such as shape blending, motion transfer and shape sequence editing.

Producing a shape sequence bridging two given shapes with large-scale deformation is still a challenge. The difficulty lies in independently determining the winding number between the corresponding local areas of the two shapes when some of the areas rotate by an angle more than $\pi$. Rotation propagation works well for planar shape interpolation [1,2], however, its extension to the 3D case is not a trivial task due to more rotational degrees of freedom. As relative transformations between adjacent triangles are usually small, shape interpolation based on connection maps [3] is promising in addressing the large-scale deformation issue. Unfortunately, 3D shapes recovered from connection maps usually suffer from shearing and shrinkage [4].

The second essential problem of shape interpolation is preserving some geometric properties during deforming process. Rigidity is one of the desired properties for the motion of objects consisting of rigid parts and several as-rigid-as-possible (ARAP) approaches have been developed to preserve this property [5,6]. Different from the rigid motion, an isometric motion allows for non-rigid deformation that keeps the geodesic distance of any two points on the surface of a deforming object [7]. Kilian et al. [8] cast the modeling of isometric motions to finding a shortest geodesic path in a high-dimensional shape space. However, their formulation is very time-consuming in optimizing the whole trajectory. Li et al. [2] propose a new as-isometric-as-possible (AIAP) formulation to attack the two problems in planar setting. However, the tricks therein lose efficacy in 3D space.

This paper develops a new framework to address the AIAP 3D shape interpolation which is not only efficient enough but also capable of coping with the large-scale deformation problem. Our key idea is modeling the isometric deformation by measuring the orthogonality between the vector of an edge and the relative velocity between its two endpoints. To support large-scale deformation, we adapt the notion of connection maps in [3] to the space of orthogonal frames defined on edges. In our framework, it is assumed that the whole sequence of bending shapes consists of triangular meshes with the same connectivity. The last pose of a sequence is traditionally obtained either by the deformation of the first pose or by cross-parametrization [9].

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The proposed approach first produces an initial shape sequence interpolating two given poses using the connection map reconstruction, and then optimizes the sequence to become an AIAP one. Our main contributions are summarized as follows:

- A 3D AIAP shape interpolation method is proposed and a framework is established based on the method for unifying motion transfer and shape sequence editing.
- A block-coordinate descent algorithm is designed to address the nonlinear AIAP optimization such that only linear equations are required to solve in each iteration; the AIAP motion of entire pose is then introduced to decouple the AIAP motions of edges. It drastically improves the efficiency of finding an optimal AIAP shape sequence. Numerical analysis shows that the algorithm converges.
- To tackle the large-scale deformation problem, connection maps between orthogonal frames of edges are introduced to initialize a shape sequence, which is a key for correctly solving the nonlinear AIAP optimization. A propagation algorithm is then devised to compute all edge frames from given connection maps as well as an orthogonal frame of a specified edge.
- The proposed initialization can easily be transplanted to other shape interpolation methods based on nonlinear optimization [8,10,2].
- Experimental results show that our method outperforms state-of-the-art interpolation methods in efficiency when visually comparable quality is achieved.

The rest of the paper is organized as follows. Section 2 presents a detailed review on related work. Theoretical background and algorithm overview are described in Section 3. An approach is introduced to generate initial shape sequences in Section 4. Formulations for shape sequence manipulations are described in Section 5. Experimental results are then discussed and analyzed in Section 6 and conclusions are drawn in Section 7.

## 2. Related work

*2D shape interpolation*: A pioneer work by Sederberg et al. [11,12] deals with interpolation of two polygonal contours. Although this approach produces pleasing intermediate results in most cases, it suffers from area distortion and edge intersection. Shapira and Rappoport [13] alleviated this issue by decomposing the object into a set of star-skeleton representations.

The as-rigid-as-possible (ARAP) shape interpolation by Alexa et al. [5] reconstructs the shape sequence by separately blending the rotational and scaling components of the transformation matrix between a pair of starting and ending triangles. Considering that the method could exhibit serious artifacts for starting and ending shapes with large-scale deformation [14], Baxter et al. devised a propagation strategy [1] to guarantee the rotation coherence among adjacent triangles. Nevertheless, both methods are sensitive to the triangulation quality of two given meshes [2]. Recently, Yang et al. extended the approach to interpolation for multi-element 2D shapes [15].

Weber and Gotsman achieved a controllable conformal interpolation by blending the representations of two given shapes [16]. A bounded distortion interpolation by Chen et al. [17] reduces the deformation distortion of triangles by blending the squared edge lengths of the given shapes. As the intermediate shapes are reconstructed using a conformal parameterization algorithm [18], their method cannot cope with the large-scale deformation problem. By viewing the interpolation as a motion problem, Li et al. introduced an as-isometric-as-possible (AIAP) framework which models the interpolation from the viewpoint of motion [2]. Besides the aforementioned approaches, there is also a great deal of work in the context of 2D shape morphing [19–21].

*3D shape interpolation*: The linear rotation-invariant method [22] represents geometric details as discrete forms based on the local frames of vertices. It first computes the geometric details of an intermediate mesh by interpolating those of the two given shapes, and then reconstructs the mesh based on linear optimization.[1] MeshIK linearly blends the deformation gradient between the start and end triangles in order to recover the intermediate poses [23]. Like the ARAP shape interpolation [5], MeshIK also performs polar decomposition on the transformation matrices to achieve correct rotation. Recently, Levi and Gotsman [5] propose a new ARAP-type energy to achieve a consistent discretization for surfaces with smooth rotation enhancement.

Poisson shape interpolation [24] reconstructs the intermediate shapes from gradient fields computed by blending those of the given shapes. Though being able to preserve surface details well, the approach does not precisely satisfy the interpolation requirement because the same Laplacian matrix is used for all shapes. Moreover, it is only suitable for interpolating shapes with small deformation.

Shape space modeling by Kilian et al. [8] accounts for 3D shapes as points of a high-dimensional Riemannian shape space equipped with an isometric inner-product. Interpolation of two 3D shapes is then converted to find a geodesic path in the shape space. Similarly, Heeren et al. [25] find a geodesic path by modeling time-discrete geodesic paths of shape sequences in the space of shells instead. Both methods result in a large-scale nonlinear optimization problem and cannot cope with the large-scale deformation problem due to linear initialization. Rossignac and Vinacua [26] defined a SAM to characterize the motion between two shapes as the power of the transform matrix.

Meshless modeling of deformation and motion [27] represents a shape as the blending of a set of nodes sampled from the volume of the shape based on a meshless finite element formulation. Keyframe interpolation is then casted into a constraint minimization on the trajectories of the nodes. Huang et al. [28] proposed a physically based interpolation method which decomposes a shape into modal coordinates.

Winkler et al. [29] propose a multi-scale interpolation scheme that hierarchically interpolates the edge lengths and dihedral angles using a multi-registration strategy [30]. Fröhlich and Botsch [10] also reconstructed the intermediate pose by specifying the length and dihedral angle of its edges. Marras et al. [31] propose an approach to efficient interpolation for articulated shapes using mixed shape spaces. Once an appropriate shape space is found, simple linear interpolation in this space produces similar results compared to [29]. However, this approach is particularly tailored for meshes that represent articulated shapes.

Example-based approaches are developed to improve the naturalness of interpolated poses. Martin et al. [32] propose strain field based approaches to balance physical correctness according to object properties from examples. Recently, Gao et al. [33] develop a data-driven approach for shape morphing to obtain more realistic results.

*Shape sequence manipulation*: To our knowledge, only the planar AIAP interpolation [2] has explored the manipulation of 2D shape sequences. It employed the relative velocity field to convey the motion information of the original shape sequence to the new shape of the edited pose for generating a new sequence. In 3D space, several editing frameworks have been developed for deforming mesh sequences. For example, Xu et al. [34] cast the edit of mesh sequences into two steps: first adjusting the

[1] Refer to the supplemental material for a private communication between the author and us.