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Isometric shape interpolation

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ABSTRACT

Acquiring natural-looking in-betweens is a fundamental shape interpolation problem in computer graphics. Several previous studies showed that as-isometric-as-possible interpolation is the key to natural and intuitive results. With this presumption, this paper describes a novel method for acquiring an isometric shape interpolation for given key-frame models. The technological and theoretical contributions of our method lie in the introduction of a new coordinate system, isometry-invariant intrinsic coordinates. This simplifies the nonlinear isometric interpolation into a simple linear algebraic problem. The method is shown to yield an effective isometric interpolation of the given key-frame models and is easy to implement. Experimental results confirm that the proposed method produces minimal metric distortion and exhibits reasonable efficiency.

1. Introduction

Shape interpolation is a technique for generating plausible shapes between two or more key-frame models. The technique is frequently used for various computer graphics applications, such as computer animation, special effects in motion pictures, and shape modeling and editing. In many applications, this technique is practiced on triangular mesh models. We assume in the following discussions that the key-frame models are already given in the form of triangular meshes with compatible topologies.

A naïve approach to shape interpolation is to linearly interpolate the coordinate values of the corresponding vertices. However, this approach has a critical defect: it is not invariant to the choice of coordinate frame, i.e., not invariant to rigid motion in the model. Further, it is not invariant to articulated motion (i.e., changes in posture). A number of studies have attempted to overcome this difficulty by aligning the positions and orientations of the models prior to interpolation [1–4]. These efforts, unfortunately, do not solve the problem completely. Indeed, it is evident from many other related works, such as Fig. 12 in [5] or a preview of our results in Fig. 1, that significant shape distortion remains despite this pre-alignment.

In one branch of interpolation research (e.g., [6–9]), the problem was formulated in terms of local rigidity preservation, assuming that the interpolation that best preserves local rigidities in the key-frame models would generate the most plausible result. On the other hand, some approaches such as [10] use an elasticity metric for the interpolation calculation. However, although such schemes return plausible results that look natural to the naked eye, metric distortions and unnatural effects remain (see, e.g., Fig. 2 of [11]). As stated in [11], it is more important to preserve the local distances rather than the local rigidity. Indeed, a number of previous works, including [11–13], suggest the use of the *as-isometric-as-possible* scheme for more natural and plausible results. Mathematically, an *isometry*, or distance-preserving mapping, is a bijection between metric spaces that preserves the distance between points. Typical examples of isometry are rigid-body transformations. In a weak sense, skin deformation caused by posture changes is also considered an isometry. Therefore, the as-isometric-as-possible scheme attempts to minimize the length distortion in an interpolated model from key-frame models.

The works mentioned above solve the interpolation problem in a somewhat computationally expensive manner, i.e., by repeatedly solving nonlinear equations through a number of iterations. Moreover, these methods are impractical for applications such as example-based modeling where many exemplar models must be interpolated, because they cannot be applied to more than two key-frame models at once.

Such problems occur because nonlinear techniques represent a shape naïvely using vertex coordinates and compute the interpolation directly from them. Instead, it is preferable to represent a shape by its intrinsic geometric properties, as this greatly simplifies the formulation of the isometric interpolation. Differential coordinates, such as those employed in [5,14–17], share a similar idea. They define a new coordinate system based on the local difference in vertex positions, normals, and other differential geometry features in the local neighborhood, and compute the linear interpolation in this new intrinsic coordinate system. Then, by reconstructing the vertex coordinates from the intrinsic values, an interpolated model can be successfully generated.

Although the focus of those differential coordinate approaches is far from the essential issue of isometric deformation, these

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0097-8493/

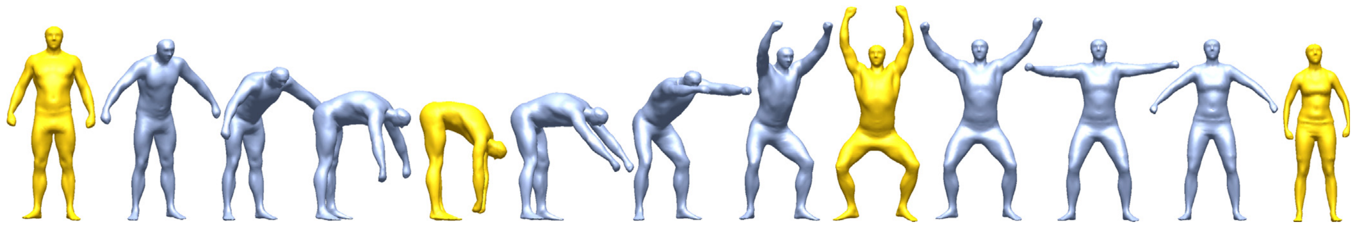

Fig. 1. Isometric interpolation of different human models. Yellow denotes key-frame models, and blue denotes the resultant in-betweens. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

approaches are valuable because they suggest the useful concept of changing the variables. Motivated by this concept, we propose a novel isometric interpolation approach based on the local intrinsic geometry. The key to our approach is to build an invertible mapping between the vertex coordinate representation and the new intrinsic coordinate system, which we call *isometry-invariant intrinsic coordinates* (IICs). In this configuration, isometric shape interpolation can easily be computed as a simple linear algebraic interpolation of coordinate values, and the corresponding geometry can be reconstructed using a rapid algorithm.

2. Isometry-invariant intrinsic coordinates

The fundamental theorem of surfaces states that two surfaces are identical if and only if their first and second fundamental forms are identical to each other [18]. Because the first and second fundamental forms concern the local metric and curvature properties of a surface, respectively, the fundamental theorem of surfaces suggests that a surface can be completely described by its local lengths and curvatures. For discrete surfaces having a graph structure, e.g., triangular meshes, this idea can be further developed to the theory that a discrete surface with a graph structure can be completely determined by its edge lengths and relative rotations between adjacent facets.

2.1. Definition

Let $\mathcal{G}=\{\mathcal{V},\mathcal{K}\}$ be a triangular mesh, where $\mathcal{V}$ is a set of $N_{\mathcal{V}}$ points, i.e., $\mathcal{V}=\{\mathbf{v}_i\in\mathbb{R}^3 \mid i=1,\ldots,N_{\mathcal{V}}\}$, and $\mathcal{K}$ is a simplicial complex containing the abstract connectivity structure. Three types of elements, namely, vertices $\{i\}$, edges $\{i,j\}$, and faces $\{i,j,k\}$, make up the abstract structure $\mathcal{K}$. Note that every tuple in $\mathcal{K}$ is an ordered collection, that is, $\{i,j\}\neq\{j,i\}$, $\{i,j,k\}\neq\{i,k,j\}$, and so on.

Before we define the IICs, let us first compose a local coordinate frame $\{\hat{\mathbf{a}},\hat{\mathbf{b}},\hat{\mathbf{n}}\}$ for each facet. The local frame is determined such that $\hat{\mathbf{n}}$ is the unit normal vector of a corresponding facet, $\hat{\mathbf{a}}$ is the unit vector parallel to the first edge of the facet, and the remaining basis vector $\hat{\mathbf{b}}$ is naturally determined from the right-hand rule.

In the configuration shown in Fig. 2, let us consider two facets, $\{r,j,i\}$ and $\{j,l,i\}$, that are both in $\mathcal{K}$ and share the same edge, $k=\{i,j\}\in\mathcal{K}$. In addition, let us denote the local coordinate frames of these facets as R and L, respectively. Further, let L' be a rotated version of L along an axis $\hat{\mathbf{e}}_k=(\mathbf{v}_j-\mathbf{v}_i)/(\|\mathbf{v}_j-\mathbf{v}_i\|)$ so that the normal directions of R and L' are the same. Then, from the geometry, the relative orientation between the facets $\{r,j,i\}$ and $\{j,l,i\}$ can be fully determined by only two scalar values, θ_k and ϕ_k, where θ_k is defined as the angle from L' to L around the axis $\hat{\mathbf{e}}_k$, and ϕ_k is the angle from R to L' around the axis parallel to the normal direction of R. Note that $-\pi\le\theta_k<\pi$, and $0\le\phi_k<2\pi$.

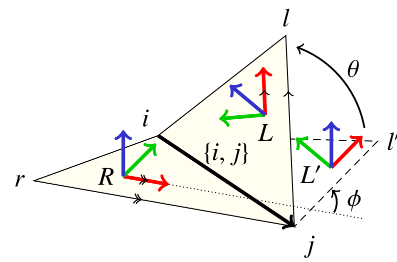

Fig. 2. Definition of isometry-invariant intrinsic coordinates (IICs) for an edge $\{i,j\}$. Basis vectors of local coordinate frames are denoted by red, green, and blue arrows, in the order $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, $\hat{\mathbf{n}}$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

According to this notion, the IICs on each edge are defined as follows:

$$\mathbf{x}_k=[l_k\ \ \theta_k\ \ \phi_k]^T, \tag{1}$$

where l_k is the edge length. Finally, the entire triangular mesh $\mathcal{G}$ is represented by an ordered collection of the IICs:

$$\mathbf{x}=[\mathbf{x}_1^T\ \cdots\ \mathbf{x}_{N_{\mathcal{E}}}^T]^T, \tag{2}$$

where $N_{\mathcal{E}}$ is the number of edges in $\mathcal{K}$.

2.2. Reconstruction of the extrinsic geometry

Although constructing the IICs from the Euclidean vertex coordinates is trivial from the definition, the inverse process is slightly more complicated. In fact, the reconstruction problem is equivalent to determining the optimal vertex positions that satisfy the given abstract connectivity structure, with its known edge lengths and relative angles between adjacent facets. Including the length and angle terms in the reconstruction process inevitably introduces nonlinearity, as they include square roots and trigonometric functions.

Indeed, some relevant studies use the edge lengths and relative angles for shape representation. On one hand, approaches such as [19,20] minimize the reconstruction error by using iterative methods. However, they are quite time-consuming because they solve nonlinear equations for each iteration step.

On the other hand, some studies such as [21,22] have solved the problem by directly reconstructing the facets propagating from a fixed facet. However, as may have been predicted, such a propagative reconstruction cannot reconstruct the surface seamlessly, as the different propagation paths return conflicting vertex positions. For this reason, these methods produce highly defective results with multiple disfigurements and artifacts, as shown in Fig. 8.

To resolve these problems, we propose a novel method of reconstructing the original vertex coordinates from the given edge lengths and relative angles in the IICs. Notably, nonlinear terms are not involved in any iteration step; hence, the algorithm is highly efficient. In addition, despite the linearity of our formulation, the results exhibit reasonable accuracy.

First, the global orientation of the local frames is computed. For this, let us recall the two facets $\{r,j,i\}$ and $\{j,l,i\}$ in $\mathcal{K}$ that share