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Blending of mesh objects to parametric surface

Lip M. Lai *, Matthew M.F. Yuen

Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

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ABSTRACT

This paper proposes a blending scheme to blend a triangular mesh and a NURBS surface together. The product is called a Hybrid PN Parametric Surface which is a watertight B-Rep surface model of a complex object. The hybrid surface is a compact model taking advantages of the simplicity of NURBS representation and detail geometric description of the triangular mesh model for sculptured objects. PN-Triangle is adopted in the mesh modeling scheme to facilitate a parametric representation. The compatibility between parametric represented mesh and NURBS surface boundaries is achieved by knot insertion. The proposed method also provides discussion on the relevant constraints in ensuring continuity across the blending boundaries between the mesh and the parametric surface. A correspondence between these boundaries is designated and an initial blending surface is generated as a transition between the boundaries to form a hybrid surface model. Then an energy minimization scheme subjected to the continuity constraints is employed for smoothing the transition such that the result is a geometrically smooth hybrid surface B-Rep.

1. Introduction

Boundary Representation (B-Rep) is often used to model objects in computer. Parametric surface such as Non Uniform Rational B-Spline (NURBS) surface is one of the most commonly used B-Rep surface models to represent free-form objects due to its concise representation scheme. However there are limitations to the complexity of the surface it can model. Triangular mesh model, as an alternative B-Rep scheme, can virtually model any complex surface. Intricate details in the model can be closely represented by increasing the mesh density. However, mesh models require significantly more storage. This results in large file size, creating an obstacle for model sharing between networked devices, particularly for wireless communication.

In designing products such as toys or jewelry items, the model would likely consist of a mixture of complex details and free-from surfaces. Using triangular mesh to build the entire computer product model would result in large data file and inhibit model sharing. It is therefore attractive to have a hybrid representation scheme that represents the basic free-form surface using NURBS surface and the complex details using mesh surface.

The method introduced in this paper allows blending of triangular mesh and Bi-cubic NURBS surface to create a new hybrid surface representation scheme. The triangular mesh and NURBS surface are blended together by creating a blending surface linking the mesh to a smooth iso-parametric boundary cut on the NURBS surface. The topology on the mesh boundary and the NURBS surface boundary has to be consistent. PN-Triangle technique is adopted to cater for smooth connection in this hybrid representation. The junction between the mesh faces and NURBS patches is guaranteed watertight and geometrically continuous.

2. Related work

Mesh can be subdivided infinitely to obtain a smooth surface. Subdivision schemes are created for mesh refinement. Catmull and Clark [1] and Loop [2] are classic subdivision schemes which work for quad and triangular mesh respectively. They work well for arbitrary topology initial mesh and preserve the topological information after the surface has been smoothed. The resulting smooth surface approximates the initial polygonal mesh and both schemes produce a surface with C^2 continuity everywhere and C^1 at extraordinary points. The other subdivision schemes are interpolation schemes include the Butterfly scheme proposed by [3,4] and the [5] scheme. An alternative method to approximate a smooth surface using triangular mesh is the curved point-normal triangles (PN-Triangles) proposed by [6]. The PN-Triangle is effectively a cubic Bezier patch that matches the point and normal information at the vertices of the flat triangle. Its normal is a Bezier interpolant of data. Fünfzig et al. [7] proposed the PNG[1]-Triangle by considering the neighborhood information of each face

* Corresponding author.
E-mail addresses: lip@ust.hk (L.M. Lai), meymf@ust.hk (M.M.F. Yuen).

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such that each pair of adjacent curved triangles share common tangent plane along the edge. There are other curved triangle methods compared in the work by [8]. Those methods are variations of Bezier patches while the PN-Triangle is the simplest and most efficient scheme and it is therefore the preferred method to represent a smooth mesh in this paper.

NURBS surface is a generalization of Bezier surface. A detailed description can be found in [9]. The major challenge using NURBS in solid modeling is finding the intersection between patches. It is a complex task and [10] proved that the intersection curve of two bi-cubic patches is of degree 324. Methods to approximate intersection curves between two patches were suggested by [11,12] but gaps do exist between patches. A generalization of NURBS is introduced by [13] called T-Spline which is a surface that can be embedded in the T-NURCC subdivision surface. The local refinement capability of T-Spline enables T-junction creation and reduces superfluous control points. The approximated intersection between patches can be filled by converting the trimmed NURBS patches to untrimmed T-Spline patches but this introduces extraordinary points as described in [14]. Manifold Spline proposed by [15] gave a concrete foundation on modeling manifold by triangular spline surfaces. It concluded that the evaluation of any planar spline is invariant to the affine transformation of its domain, and it is possible to be a generalized manifold domain of arbitrary topology. Later, [16] proposed the manifold T-Spline.

Hybrid modeling catches the attention of researchers since a hybrid modeling scheme benefits from mixing the different representations. Adzhiev et al. [17] studied a hybrid system of volumetric data, voxel, and implicit surface functions. The method depends on the conversion between voxels and implicit surfaces. Constructive solid geometry (CSG) has been extended in the work of [18] by using a hybridized CSG tree structure called The HybridTree. Blending or Boolean operations between implicit surfaces and meshes are dependent on the associated field functions. The NURBS-based hybridize element proposed by [19] was used to solve the compatibility problem between element surfaces in FEA and CFD simulations. Specific hybrid elements with mesh elements joining a NURBS surface with a watertight boundary was proposed by [20] but the method does not offer a generic hybrid element definition. Martin et al. [21] mixed mesh elements and NURBS elements in volumetric models as a kind of hybrid structure, but the gaps were filled only by an infinite refinement on the mesh side. Modeling free form solid by using Extended Simplicial Chains and PN-Triangles was studied by [22]. The solid resulting from Boolean operation contained trimmed patches.

In summary, there is no efficient hybrid modeling technique to model objects with a mixed mesh and parametric surface using standard B-Rep representation. Existing methods generate a hybrid surface without the guarantee of forming a watertight model and the holes are filled explicitly. This affects the robustness of subsequent operations, e.g. Boolean, that heavily rely on a valid B-Rep model. In addition, the lack of smoothness definition between mesh and parametric surfaces leads to undetermined continuity requirements. To address these issues, e.g. to generate a surface connecting the inputs shown in Fig. 1, the proposed representation, called Hybrid PN Parametric Surface, provides a watertight surface model containing a mixed mesh and NURBS surface. However, the mesh and NURBS surface do not share a common parametric representation; therefore creating a watertight connection is a challenge. The first step in resolving this challenge is presented in Section 3 by changing the triangular mesh to PN-Triangle Mesh (PN-Mesh) representation and a NURBS curve representing the exact PN-Mesh boundary is also created. Continuity for smooth connection between mesh and NURBS surface with prescribed boundary conditions is achieved by a cubic NURBS transition surface presented in Section 4 that guarantees geometrically smooth connection. The NURBS curves representing the NURBS surface and the PN-Mesh boundaries should have

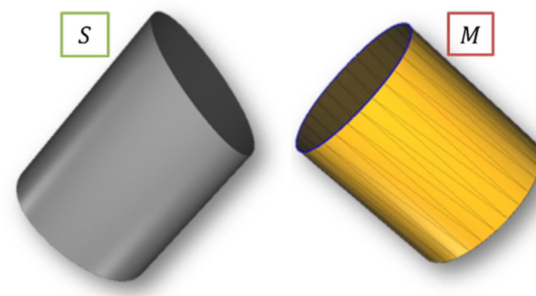

Fig. 1. Given inputs. (Left) NURBS surface *S* 'open-ended inclined tube'. (Right) Triangular Mesh *M* 'open-ended tube'.

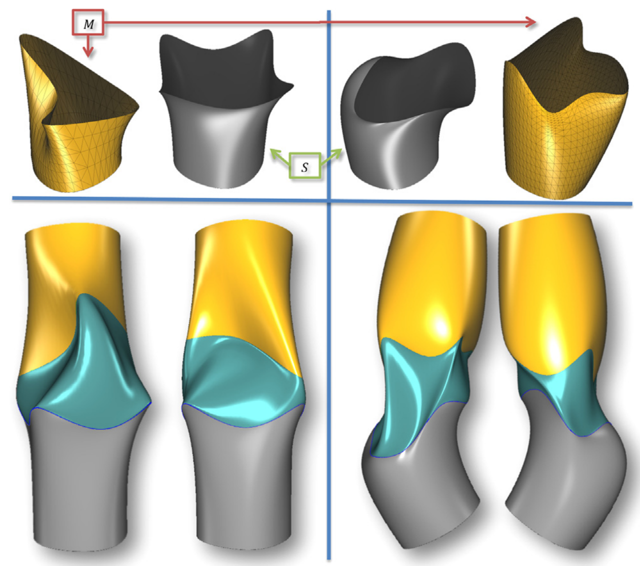

Fig. 2. Open freeform meshes and NURBS surfaces with freeform boundaries. The upper part shows the inputs and lower part shows the front and back views of the results. The PN-Meshes are shaded in yellow. The input parametric surfaces are shaded in grey while the generated transition surfaces is shaded in cyan. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

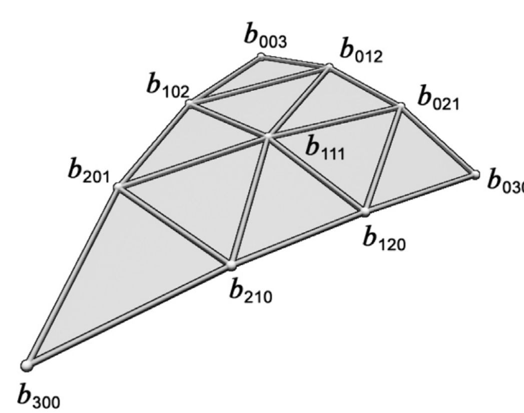

Fig. 3. The control net of triangular Bezier patch after [6].

compatible knot vector to ensure proper connection. The smoothness of the transition surface is guided by minimizing the thin plate energy function of the surface as presented in Section 5. The proposed Hybrid PN Parametric Surface in this paper is smooth and watertight. It is a heterogeneous representation of meshes and NURBS surfaces without the introduction of extra information. The key limitation of the