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Technical Section

A sample-based method for computing the radiosity inverse matrix ☆

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The radiosity problem can be expressed as a linear system, where the light transport interactions of all patches of the scene are considered. Due to the amount of computation required to solve the system, the whole matrix is rarely computed and iterative methods are used instead. In this paper we introduce a new algorithm to obtain an approximation of the radiosity inverse matrix. The method is based on the calculation of a random sample of rows of the form factor matrix.

The availability of this matrix allows us to reduce the radiosity calculation costs, speeding up the radiosity process. This is useful in applications where the radiosity equation must be solved thousands of times for different light configurations. We apply it to solve inverse lighting problems, in scenes up to 170 K patches. The optimization process used finds optimal solutions in nearly interactive times, which improves on previous work.

1. Introduction

The radiosity method is a global illumination solution that is extensively used in rendering applications and lighting design. The basic algorithm is based on the finite element approach, assuming that the light is reflected diffusely by the surfaces. In the classical approach, the explicit computation of the radiosity matrix is avoided due to the huge computational cost. One of the early formulations proposed is the progressive refinement solution [1], where a selection of form factors is computed, belonging to the most significant patches that must "shoot" their accumulated radiosity. Another important contribution is the hierarchical radiosity method [2], which reduces the $O(n^2)$ form factor calculations to $O(n)$, for a scene composed of n patches. These methods were successful at computing the global illumination of a scene. Further developments resulted in interactive techniques that are currently used in architectural lighting design applications and in visualization software.

In most cases, neither the full radiosity matrix nor its inverse is required to compute a 3D scene, and few number of iterations is enough to approximate the radiosity solution. However, the inverse of the radiosity matrix, expressed as $\mathbf{M} = (\mathbf{I} - \mathbf{RF})^{-1}$ [1], is useful when the illumination of the scene must be calculated a huge number of times. In this equation, $\mathbf{I}$ is the identity matrix, $\mathbf{R}$ is a diagonal matrix containing the surface reflectivities, and $\mathbf{F}$ is the form factor matrix (Table 1).

One kind of problems that requires a huge number of global illumination calculations is the inverse lighting problem (ILP). Unlike traditional direct global illumination calculations, in ILP the surfaces have lighting goals and constraints to satisfy, and the shape, position and power of emitters are the variables of the problem. We follow the notion that ILPs are formulated for static geometries, as laid down by Marschner [3].

These problems could be formulated as optimization problems, which usually are implemented as iterative algorithms where at least one radiosity calculation is performed in each iteration. Solving ILP is a challenge since the global illumination computation should be solved for thousands of possible configurations of the emitters. One of the limitations observed in previous approaches of ILP is that they are only feasible for managing simple scenes.

We present results obtained from scenes composed up to 170 K triangles. As it happens with any 3D model, the fine grain scene can be simplified into a rough approximation using standard polygon reduction techniques or clustering (Fig. 1(a) and (b)).

Instead of doing that, our proposal is based on the random selection of a small set of patches $\mathcal{P}$, and the calculation of the form factors between those patches and all the scene patches.

Our method generates a "sample-based scene", or $\mathcal{S}_{\mathcal{P}}$ (Fig. 1(c)). This method facilitates the calculation of an approximate representation of $\mathbf{M}$. A small selection of patches provides enough information to calculate a good approximation of the radiosity

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values. This is mainly caused by the spatial coherence implicit in the radiosity method.

If we were to build **M** based on a rough approximation of the scene, there would be three key issues to solve. One issue is how to simplify the geometry when we try to limit the error of the global illumination of the scene. A second issue is the difficulty in reducing the number of patches to a few thousands, when the scene contains many arcs, windows, statues, furniture and other architectural elements. Finally, another issue is the mapping of the solution into the original model. However, the use of a sample-based scene avoids the treatment of the key issues, facilitates the exploration of many possible configuration of $\mathcal{S}_{\mathcal{P}}$, and makes possible to estimate the error of the radiosity values.

Fig. 1(c) shows a schematic representation of the information managed. In this example, a sample-scene composed of eight patches is taken and the form factors between the sample and the rest of the scene are computed to obtain the rows of **F** (Fig. 2). Based on the symmetry of **AF**, where **A** is a diagonal matrix containing the area values, the columns of **F** can be calculated.

The information extracted from the sample is combined through the use of operators that process the light interactions in the scene. In Section 3 a generic definition of each operator is presented, and in Section 4 a specific definition for them is proposed. The main contributions of this paper are the following:

- A description of the radiosity inverse matrix **M** using operators and a sample-based scene.
- A matrix-based implementation of the operators.
- A method to estimate the amount of patches in the sample-based scene.
- A significant speed up in the resolution of inverse lighting problems for medium size models.

The rest of the paper is divided into six sections. In Section 2 we describe previous work. Sections 3 and 4 are dedicated to explain the details of the method proposed. In the following section we analyze the error estimation and an estimation of the sample size. Section 6 describes the experimental results, and finally the conclusions and further work are summarized in Section 7.

Table 1
Symbol notation and meaning.

L	Set of all potential light emitters
l_i	Emitter i
p_i	Patch i
$\mathcal{P}$	$\{p_1 \cdots p_{\lvert\mathcal{P}\rvert}\}$; sample of $\mathcal{S}$; $\mathcal{P} \subset \mathcal{S}$
$\mathcal{P}_L$	$\{l_1 \cdots l_{\lvert\mathcal{P}_L\rvert}\}$; sample of L; $\mathcal{P}_L \subseteq L$
$\mathcal{S}$	Set of all scene patches
$\mathcal{S}_{\mathcal{P}}$	Sample-based scene
u	Surface; $u \subseteq \mathcal{S}$
n	$\lvert\mathcal{S}\rvert$; number of patches in $\mathcal{S}$
$\mathbf{M}_{\mathcal{S}}$	Radiosity operator for the scene $\mathcal{S}$
$\mathbf{P}_{u_2 \Leftarrow u_1}$	Projection operator from u_1 to u_2
$\mathbf{S}_{b \Leftarrow a}$	Selection operator from a to b
$A_{\mathcal{S}}$	Area of the scene $\mathcal{S}$
B	Radiosity in the entire scene
$\widetilde{B}$	Approximation of the radiosity values
B_u	Total radiosity in surface u
B_u^D	Direct radiosity in surface u
B_u^I	Indirect radiosity in surface u
$\phi^{(\eta)}$	$\{\widetilde{B}^{I(1)} \cdots \widetilde{B}^{I(\eta)}\}$;sample of $\widetilde{B}^I$
E	Emission
ε, $\mathcal{E}$	Error
$r_{ff}(i)$	$(\sum \text{all } ff)/(\sum \text{sampled } ff)$, $\forall ff \in \mathbf{F}(i,:)$
$\mathbf{A}$	Diagonal matrix with area values of $\mathcal{S}$
$\mathbf{A}_{\mathcal{S}_{\mathcal{P}}}$	Diagonal matrix with area values of $\mathcal{S}_{\mathcal{P}}$
$\mathbf{F}$	Form factors matrix
$\mathbf{F}_{\mathcal{S}_{\mathcal{P}}}$	Form factors matrix of $\mathcal{S}_{\mathcal{P}}$
$\mathbf{I}$	Identity matrix
$\mathbf{M}$	Inverse of the radiosity matrix
$\widetilde{\mathbf{M}}$	Low-rank approximation to $\mathbf{M}$
$\mathbf{R}$	Diagonal matrix with reflectivity indexes
$\mathbf{Y}$, $\mathbf{V}$	Components of $\widetilde{\mathbf{M}}$ ($n \times k$ matrices, $n \gg k$)
μ	Mean
$\overline{\mu}_A$	Sample mean, weighted by the areas
σ	Standard deviation
$\overline{\sigma}$	Sample standard deviation
LTI	Length of the tolerance interval

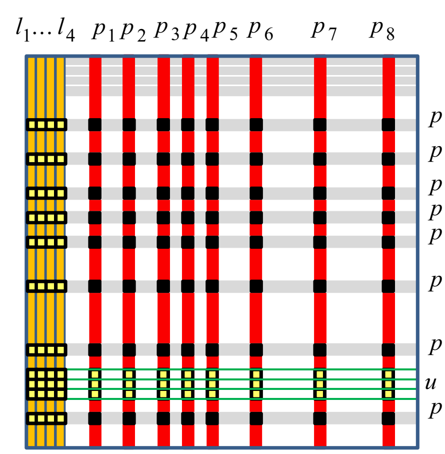

Fig. 2. Sample rows/columns of **F**.

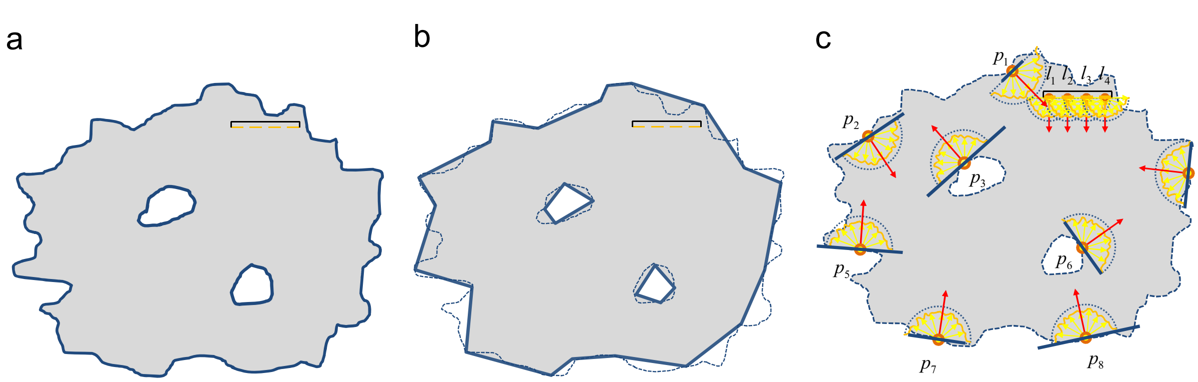

Fig. 1. Three different representations of a 2D projected scene. (a) Fine grain scene $\mathcal{S}$, (b) Coarse grain scene and (c) A sample-based scene ($\mathcal{S}_{\mathcal{P}}$).