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A revisit to fitting parametric surfaces to point clouds ☆

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ABSTRACT

We study the performance of algorithms for freeform surface fitting when different error terms are used as quadratic approximations to the squared orthogonal distances from data points to the fitting surface. We review the TD error term and the SD error term in surface fitting to point clouds, present robust surface fitting algorithms using the TD error term and a new variant of the SD error term. We report experimental results on comparing them with the prevailing PD error term in the setting of fitting B-spline surfaces to point cloud data. We conclude that using the TD error term and the SD error term leads to surface fitting algorithms that converge much faster than using the PD error term.

1. Introduction

Freeform parametric surface fitting to point clouds is a fundamental problem in computer aided design and computer graphics. There are many aspects to address toward an automatic algorithm for fitting a parametric surface to noisy point clouds [15,16]. This problem has been studied extensively in the literature and many approaches have been proposed [3,4,6–9,14,15].

We are interested in fitting a parametric surface S to a point cloud X by minimizing the *squared orthogonal distance* from X to S which leads to a nonlinear least squares minimization problem. We use B-spline surfaces as an instance in this paper but our discussions also apply to other parametric surfaces, such as subdivision surfaces. Let $X=\{X_i, i=1,\ldots,m\}$ be a set of data points, forming a target surface shape. Let $S(u,v,P)$ denote a B-spline surface, where $P=\{P_0,\ldots,P_n\}$ are control points of S; u and v are parameters of S. We fix the values of knot vectors and only the control points are variables to solve. The problem of surface fitting to a point cloud X is defined by

$$\min_P \frac{1}{2}\sum_{i=1}^{m} d^2(S,X_i)+\lambda f_s, \qquad (1)$$

where $d(S,X_i)$ defines the orthogonal distance from X_i to $S(u,v)$, i.e., $d(S,X_i)=\|X_i-S(u_i,v_i)\|$, where $X_i-S(u_i,v_i)$ is orthogonal to the tangent plane at $S(u_i,v_i)$. It is clear that the distance function d is nonlinear in P since the foot point $S(u_i,v_i)$ of X_i is a nonlinear function in P. The function f_s, a quadratic function in P, defines the fairness of S which is commonly used for maintaining surface fairness and stabilizing surface fitting procedure [15].

The above nonlinear least squares minimization problem is usually solved by iterative methods that consist of the following steps.

1. *Initialization*: Construct a surface S roughly close to the point cloud X as an initialization.
2. *Foot point computation*: Compute the closest point on S for each target point X_i.
3. *Control points update*: Minimize a quadratic function to update the fitting surface S.
4. *Termination*: The iteration is terminated when the fitting error is smaller than a threshold, otherwise go to step 2.

In our algorithm, the initial B-spline surface is provided manually by specifying the number of control points, the initial positions of control points and the values of knot vectors. In this paper we are particularly interested in the convergence rates of surface fitting algorithms when different error terms are used. We stress that we do not intend to propose a fully automatic algorithm, which would entail the automatic determination of the initial surface and progressive update on the number of control points, as done for B-spline curve fitting in [17].

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It is known that the step of control point update determines the convergence rate of a B-spline curve fitting algorithm [14]. In this step, a sum of error terms is minimized by the solution of a linear equation system [5,2]; that is, we solve

$$\min_{P} \frac{1}{2}\sum_{i=1}^{m} e(S,X_i)+\lambda f_s, \tag{2}$$

for the control points of an updated surface. The error term function $e(S, X_i)$ is an approximation to the nonlinear function $d^2(S,X_i)$. This quadratic function in the control points of an unknown surface has a great influence on the performance of the surface fitting algorithm.

Recently, the TD error term and SD error term have been proved more efficient than the prevailing PD error term for B-spline curve fitting to point clouds [14] and in some specific surface fitting applications [9]. However, the performances of these two error terms have not been demonstrated for B-spline surface fitting to point clouds. We will conduct an experimental study of the performance of surface fitting algorithms using the TD error term and a variant of the SD error term and show that they lead to surface fitting algorithms that converge much faster than a method using the prevailing PD error term. In the following, the surface fitting algorithms using the PD, TD and SD error terms will be called the PDM, TDM and SDM methods, respectively.

2. Error terms in surface fitting to point clouds

In the following discussions, we use $S_c(u,v)$ to denote a surface $S(u,v,P_c)$. We use $S_+(u,v)$ to denote an updated surface $S(u,v,P_+)$, where $P_+ = P_c+D$ are control points of the updated surface and D are the incremental updates to the current control point P_c. Therefore, D are the unknown variables to solve in one iteration. From the geometry viewpoint, an error term $e(X_i,S)$ defines the squared distance from X_i to a local approximation of the unknown fitting surface at $S_+(u_i,v_i)$ where (u_i, v_i) are parameters of the foot point of X_i on $S_c(u, v)$. A faithful quadratic approximation to the true squared orthogonal distance requires a faithful local approximation to the fitting surface $S_+(u,v)$, which should depend on local curvatures of the shape to approximate.

2.1. PD error term

The PDM method is the most widely used method for surface fitting due to its simplicity. The PD error term is defined by the squared distance from a target point X_i to a point on the surface $S_+(u,v)$ at a particular parameter

$$e_{PD,i}(D) = (X_i - S_+(u_i,v_i))^T(X_i - S_+(u_i,v_i)),$$

where (u_i,v_i) are the parameters of the foot point of X_i on $S_c(u,v)$. Considering the fact that the fitting surface is a variable surface and the parameters u_i and v_i are also functions of P_+, the PD error term is inaccurate to approximate the true squared orthogonal distance.

2.2. TD error term and its regularization

The TD error term for surface is defined by the squared distance from a point X_i to the tangent plane at its foot point on $S(u,v)$

$$e_{TD,i}(D) = [(X_i - S_+(u_i,v_i))^T N_i]^2,$$

where N_i is a unit normal vector at $S_c(u_i,v_i)$. The TD error term uses a plane as a local approximation to a surface which is inaccurate at surface regions of high curvatures. Consequently, the TDM method has unstable behaviors at surface regions of high curvatures. The Levenberg–Marquardt method has been suggested to be combined with the TDM method to improve its convergence behaviors [12]. With the Levenberg–Marquardt method, the quadratic function to minimize in each iteration becomes

$$\frac{1}{2}\sum_{i=1}^{m} e_{TD,i}(D)+\mu\|D\|^2+\lambda f_s. \tag{3}$$

The minimizer of function (3) is the solution of a linear equation

$$(A+\mu I)D = b, \tag{4}$$

where A is a matrix corresponding to the first and the third part in (3), I is the identity matrix. The algorithm in [10] is often used to decide the value of μ in (4). TDM combined with the Levenberg–Marquardt regularization method leads to the TDMLM method.

2.3. SD error term and a modification

Pottmann et al. introduce a curvature-dependent term in curve and surface fitting for minimizing the squared orthogonal distance from sampling points on a fitting surface to the target shape [13]. A variant version of this error term, namely the SD error term, is proposed for B-spline curve fitting to point clouds in [14]. The formulation of its extension of the SD error term to surface fitting is available in [8,9]

$$e_{SD,i}(D) = \frac{d}{d+\rho_{i,1}}[(X_i - S_{+,i})^T T_{i,1})^2 + \frac{d}{d+\rho_{i,2}}((X_i - S_{+,i})^T T_{i,2})^2 + ((X_i - S_{+,i})^T N_i]^2, \tag{5}$$

where $d = \|X_i - S_c(u_i,v_i)\|$ is the distance between X_i and its foot point on current surface. $\rho_{i,1}$ and $\rho_{i,2}$ are absolute values of the principal curvature radii at $S_c(u_i,v_i)$. $T_{i,1}$ and $T_{i,2}$ are unit vectors along the principal curvature directions at $S_c(u_i,v_i)$, corresponding to $\rho_{i,1}$ and $\rho_{i,2}$ respectively. N_i is the unit surface normal vector at $S_c(u_i,v_i)$. Eq. (5) depends on the principal curvatures at $S_c(u_i,v_i)$ and is therefore a better approximation than those in the TD error term and the PD error term.

In the following, we will present a new version of SD error term which is called the SSD error term. The SD error term (5) may reduce to the TD error term when the fitting surface is flat. Due to robustness consideration as well as the aim to avoid computing principal curvatures on the fitting surface in every iteration, we use curvature information from the point cloud X in the error term, instead of using curvature information of the fitting surface. This is more reasonable since the data points represent the target shape to fit. In a preprocessing step, the principal curvature values (c_1 and c_2) at every data point X_i are estimated. For stability reasons, we use the curvature value corresponding to the principal direction which is more curved, i.e., we use an absolute value $\rho = 1/\max\{\|c_1\|,\|c_2\|\}$. Substituting $\rho_{i,1} = \rho_{i,2} = \rho$ into Eq. (5), we obtain

$$e_{SSD,i}(D) = \frac{d}{d+\rho}\{[(X_i - S_{+,i})^T T_{i,1}]^2 + [(X_i - S_{+,i})^T T_{i,2}]^2\} + [(X_i - S_{+,i})^T N_i]^2. \tag{6}$$

Note that, since $T_{i,1}$, $T_{i,2}$ and N_i are mutually orthogonal unit vectors, we have

$$e_{PD,i}(D) = \|X_i - S_{+,i}\|^2 = [(X_i - S_{+,i})^T T_{i,1}]^2 + [(X_i - S_{+,i})^T T_{i,2}]^2 + [(X_i - S_{+,i})^T N_i]^2.$$

Therefore, Eq. (6) can be rewritten as

$$e_{SSD,i}(D) = \frac{d}{d+\rho}e_{PD,i}(D) + \frac{\rho}{d+\rho}e_{TD,i}(D), \tag{7}$$

which is a weighted combination of the PD error term and the TD error term, with the weights depending on surface curvatures.