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Technical Section

Extracting a polyhedron from a single-view sketch: Topological construction of a wireframe sketch with minimal hidden elements

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ABSTRACT

An essential prerequisite to construct a manifold trihedral polyhedron from a given *natural* (*or partial-view*) *sketch* is solution of the "wireframe sketch from a single natural sketch (WSS)" problem, which is the subject of this paper. Published solutions view WSS as an "image-processing"/"computer vision" problem where emphasis is placed on analyzing the given input (natural sketch) using various heuristics. This paper proposes a new WSS method based on robust tools from graph theory, solid modeling and Euclidean geometry. Focus is placed on producing a minimal wireframe sketch that corresponds to a topologically correct polyhedron.

1. Introduction

The subject of this paper is related to 3D geometric modeling and to sketch-based CAD, with an emphasis on *natural* (*or partial-view*) *sketch* (Fig. 1(a)), i.e., a sketch without any hidden lines. More specifically, this work deals with the problem of automatically constructing a "polyhedron from a single natural sketch (PSS)". The principal sub-problem of PSS is topological construction of a "wireframe sketch from a single natural sketch (WSS)", for which a solution is presented here.

Regarding published research on the PSS/WSS problem, one observes the following: numerous papers have appeared from 1980 until today on this problem, offering various solutions; see, e.g., [1–9] and references therein. Despite that, it is fair to say that existing solutions are far from satisfactory, as, even in 2008, new methods are appearing (see, e.g., [10] and references therein) for the plainest case of the PSS/WSS problem, where the polyhedron (to be constructed) has only planar faces and is trihedral. Current methods view PSS/WSS as "image-processing" or "computer vision" problems, where emphasis is placed at analyzing the given input (sketch) using tools and techniques lacking a robust mathematical foundation. More specifically, many published methods are based on the "line-labeling (LL) methodology" (initiated in [11–13]), which tries to associate each sketch line to a "label" (with possible values = "convex", "concave", "occluding") so that the whole set of labels is compatible with a predetermined set of "labeling rules". Numerous authors have explored this idea (see [5,14–18] and references therein). After presenting, in a series of papers (see references in [17]) improved versions of the methods in [5], Varley et al. conclude [17] that LL has very little to offer towards solution of the PSS/WSS problem.

It must be emphasized that the predominant "image-processing"/"computer vision" methodologies not only place focus on poor heuristics like LL, but also they clearly place PSS/WSS out of the correct context, which obviously is "graph theory" (for analyzing the sketches) and "3D solid modeling" (for constructing the corresponding polyhedron). Recently, Cao et al. [10] published a paper on the PSS problem that indeed moves away from the classical "image-processing" methodology and adopts "graph theory" to solve the WSS problem. The proposed method includes two main steps: (a) construction of an initial hidden structure, and (b) reduction of this structure to the "most plausible one" according to "human visual perception" (these two main steps correspond to Steps 3 and 4 in the description given in Section 5.1. of [10]). Unfortunately, this method is far from being complete as both steps (a) and (b) are based on heuristic criteria/processes lacking a solid justification, e.g., realization of step (b) is solely based on the heuristic criterion "the human visual system … tends to interpret a figure in such a way as to produce an object that is as symmetrical as possible" (see Section 5.4 in [10]). This approach is problematic as it cannot handle objects that are far from symmetric; indeed the test sketches used in [10] are either symmetric or "almost symmetric".

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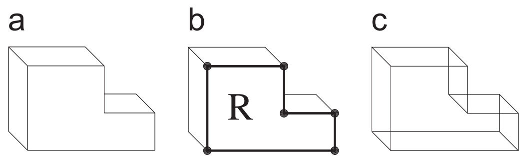

Fig. 1. (a) A natural sketch, (b) R is a region of the natural sketch/plane graph, and (c) a wireframe sketch corresponding to the plane sketch in Fig. 1(a).

The present research aims at producing a PSS algorithm that is free from the shortcomings of the "image-processing" methodology as well as of a technique like [10] based solely on "graph theory". Indeed, an improved WSS algorithm is presented here employing robust tools from graph theory, 3D solid modeling and Euclidean geometry. A detailed topological analysis of sketches is given, followed by an efficient technique to complement a given "visible sketch" with appropriate hidden parts. The scope of the present research covers manifold polyhedra without holes, which are also trihedral. The employed sketch is a natural sketch, where:

1. No element of the natural sketch causes two or more visible regions of it to correspond to one visible region in the wireframe sketch to be constructed.
2. At most one T-junction (: this is defined in Section 2) exists in each region of the sketch.

The proposed methodology aims at producing a polyhedron, which is CAD-usable, i.e., a valid 3D solid model. This objective, combined with the fact that no information is available regarding the hidden part of the polyhedron, leads to the conclusion that this hidden part should be minimal (e.g., a single planar face) and at the same time sufficient to define a valid solid model. This "minimal-completion strategy" implies that from the given natural sketch a wireframe sketch should be derived where the number of hidden lines/junctions/regions is as small as possible.

2. Geometric modeling of sketches and solids with an emphasis on topological description

A sketch is a set of straight lines on a plane that intersect at junctions (i.e., points). In current research [5,8,10,19] a sketch is considered to depict an orthographic projection of a manifold *trihedral solid* (each vertex of the solid belongs to exactly three faces) with planar faces. The solid (polyhedron) is considered to be in "general position" with respect to the given projection plane, i.e., no face or edge of the solid is perpendicular to that plane. Adjacent faces (edges) of a solid lie on distinct planes (curves).

The user draws a *natural* (*or partial-view*) *sketch* (Fig. 1(a)), i.e., a sketch without hidden lines, in the *most informative view*: this means that there is nothing at the "back of the sketch" that cannot be directly inferred from the visible part of it. Loops of lines and junctions form regions of the sketch; see an example of a region in Fig. 1(b). In this work, a natural sketch is modeled as a plane graph and graph theory is employed to produce a host of theoretical and algorithmic tools to support the "sketch-to-solid transformation".

Definition 1. A *valid wireframe sketch* (Fig. 1(c)) is a graph resulting from a given natural sketch to which hidden lines, hidden junctions and hidden regions are added, so that

1. Whenever a hidden line intersects a visible line at a point not identical with a junction of the given natural sketch, this point is not considered to be a junction of the wireframe sketch.
2. Every junction is adjacent to three lines; i.e., the degree of each junction j is $d(j) = 3$ [20].
3. Every line is adjacent to two regions.
4. The sketch is a connected graph [20].
5. Two adjacent regions of the sketch share exactly one line or two-or-more collinear lines [21].

The lines, junctions and regions of a sketch are called *elements of this sketch*. Respectively, the *elements of a solid* are its vertices, edges and faces. We note that the hidden elements of a sketch are indicated by a subscript "h", while the visible elements by a subscript "v".

The problem to be solved is derivation of a manifold trihedral polyhedron whose projection onto the sketch' plane is identical with the constructed wireframe sketch. A "one-to-one correspondence" exists between lines/junctions/regions of the wireframe sketch and the edges/vertices/faces (respectively) of the corresponding polyhedron. The method presented below assumes that there are no elements of the natural sketch causing two or more visible regions of it to correspond to one visible region in the wireframe sketch to be constructed.

A wireframe sketch is produced from a given natural sketch, when to the latter's boundary elements (elements adjacent to both the interior and the exterior of the sketch [20]) appropriate hidden lines and junctions are added. There are three types of boundary junctions in a natural sketch, according to their degree (see Fig. 2):

(a) Boundary junctions of $d(j) = 3$ (see junction j in Fig. 2(a)): Each one belongs to two visible regions and one hidden, and it is adjacent to visible lines only.
(b) L-junctions (see junction j in Fig. 2(b)): Each one is of degree 2. It belongs to one visible region, two hidden regions, and it is adjacent to two visible lines and one hidden.
(c) T-junctions (see junction j_T in Fig. 2(c)): Although a junction of this kind is drawn as a junction of degree three, it implies depth and its true location is in the hidden part of the object [5,17]. Thus, a T-junction is a junction of degree 1, since it is adjacent to one visible line and to two hidden lines. The two collinear lines adjacent to a T-junction are considered as one *occluding line* (see line e in Fig. 2(d)), which belongs only to the *occluding region* (region R in Fig. 2(d)). A T-junction belongs to two (one) hidden regions and to one (two) *partially visible regions* (see the bold-marked regions in Fig. 2(e)). It should be emphasized that, in the methods proposed here, partially visible regions are always included in the set of visible regions. The hidden region adjacent to the occluding line is called *occluded region*.

It is noted that since PSS with T-junctions is still an unsolved problem, we consider in this paper the simplest case, *where at*

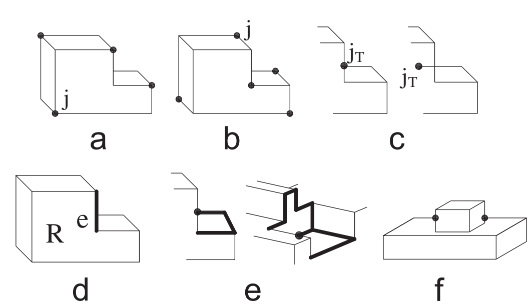

Fig. 2. (a) The junction j is a boundary junction with degree = 3, (b) the junction j is an L-junction, (c) T-junction j_T and its true position, (d) the line e is an occluding line and belongs to the occluding region R, (e) the bold-marked regions are partially visible regions, and (f) two T-junctions in a region.