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A graph-based method for fitting planar B-spline curves with intersections

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Abstract

The problem of fitting B-spline curves to planar point clouds is studied in this paper. A novel method is proposed to deal with the most challenging case where multiple intersecting curves or curves with self-intersection are necessary for shape representation. A method based on Delauney Triangulation of data points is developed to identify connected components which is also capable of removing outliers. A skeleton representation is utilized to represent the topological structure which is further used to create a weighted graph for deciding the merging of curve segments. Different to existing approaches which utilize local shape information near intersections, our method considers shape characteristics of curve segments in a larger scope and is thus capable of giving more satisfactory results. By fitting each group of data points with a B-spline curve, we solve the problems of curve structure reconstruction from point clouds, as well as the vectorization of simple line drawing images by drawing lines reconstruction.

Keywords: Curve fitting; Curve reconstruction; B-spline; Point cloud

1. Introduction

2D shapes represented by unorganized data points are frequently encountered in numerous applications, where data points are obtained using scanning devices or extracted from digital images. Unorganized data points (point clouds) are however not a suitable representation for geometric processing. Therefore, converting point clouds into parametric representations such as polylines or B-spline curves is highly demanded in many applications.

In this paper, we study the problem of computing a set of disjoint parametric curves with possible intersections from a point cloud. This problem of *curve extraction* has numerous applications in reverse engineering, where B-spline parametric curves/surfaces are required. Extracting shapes from data points as spline curves also enjoys important applications in digital image processing if the content of an image is a 2D shape composed of curves, such as line drawing images, blueprints and hand written characters. Converting digital images of line drawings into B-spline curves is also a special instance of image vectorization.

A reasonable scheme to this problem is computing meaningful groups of data points together with polyline curves. The latter can be refined by a curve fitting approach. The key point however is to identify combination of curve segments joining at intersections to form continuous curves passing through each other. The main difficulties include (1) the intersection region often contains much noise and thus may give disturbing information for recovery of intersecting curves and (2) tangential lines of joining curves near intersection regions are not sufficient for determining curve joinings. Refer to Fig. 1(a) for an intersection part of a noisy point cloud; Fig. 1(b) shows three joining curve segments and estimated tangent lines. We see that it is hard to give correct merging of curves using only tangent lines.

In this paper, we propose a framework for fitting B-spline curves to a point cloud, where the curves may intersect with each other and curves with self-intersection are also allowed. The proposed approach consists of two phases. The first phase divides the point cloud into a set of groups of data points where each group of data points represents a curve shape.

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This step recovers the topology information of data points. The second step reconstructs a B-spline curve fitting to each group of data points. This step is a geometry recovery step. The main contributions of this paper include

- A unified framework for handling noise, outliers and curve components of a point cloud.
- A graph based method to identify pairs of curve segments which can be merged into a single curve.

2. Related work

2.1. Curve reconstruction

Curve reconstruction from point clouds with or without noise is a well studied problem. Various aspects of curve reconstruction have been addressed including robustness to noise, handling of outliers and feature preservation. Numerous techniques have been employed such as Voronoi diagram, spectral analysis, image processing and optimal transport. Optimal transport is used to reconstruct 2D shapes with polyline structures which performs well for data points with shape features and large noise [1]. A large body of methods makes use of voronoi diagram to deal with curve reconstruction from a set of sampling data points. The advantage of this class of methods is that their accuracy can be proved if an appropriate sampling density is satisfied [2]. Amenta et al. proposed the crust method which utilized the β skeleton and voronoi diagram [3,4]. Wang et al. proposed a curve reconstruction method based on circular neighboring projection and normal-based smoothing [5]. However, these methods do not give a segmentation of data points. The reconstruction result is a structure of curves instead of independent curves.

Some recent works discuss recovering multiple curves or curves with self-intersections from point clouds [6–8]. Ardeshir proposed a method for grouping and fitting multiple curves to point clouds with relatively simple shape [9]. Furferi et al. presented a method for fitting weighted B-spline curves using PCA analysis [10]. Yan et al. proposed a method for curve fitting based on the fuzzy C-means clustering method [11]. Zhao et al. presented a method for fitting non-simple curves using skeleton extraction and refinement [12]. Our proposed method is different to these methods in that we make use of a skeleton representation of point cloud which provides more information on curve shape and topological relationship of curves.

2.2. Curve fitting

Curve fitting mostly refers to the problem of parametric curve approximation to noisy data points. Most existing works assume that the data points come from a single curve without self-intersections. A fitting process starts with an initial curve which is updated by minimizing some objective function measuring fitting quality and curve fairness.

Existing works discuss this problem from various aspects. A parametric curve is often used for shape reconstruction of a point cloud. Levin proposed the moving least squares method (LMS) for curve fitting [13]. Lee discussed the deficiencies of direct application of MLS in curve fitting and proposed some improvements by introducing the Minimal Spanning Tree (MST) in a pre-processing phase [14]. Some works on curve fitting focus on the fitting speed of minimizing squared

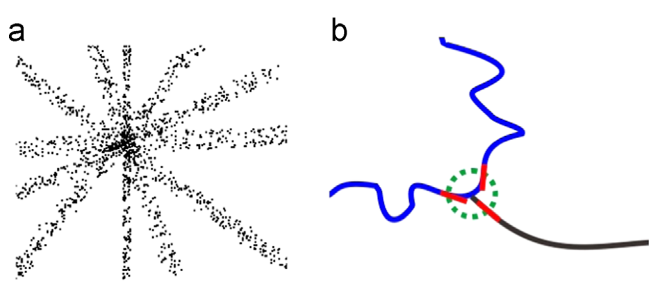

Fig. 1. (a) There is large noise at intersection regions of crossing curves. (b) Tangent vectors of meeting curves are insufficient to decide their role as curve parts of large curves.

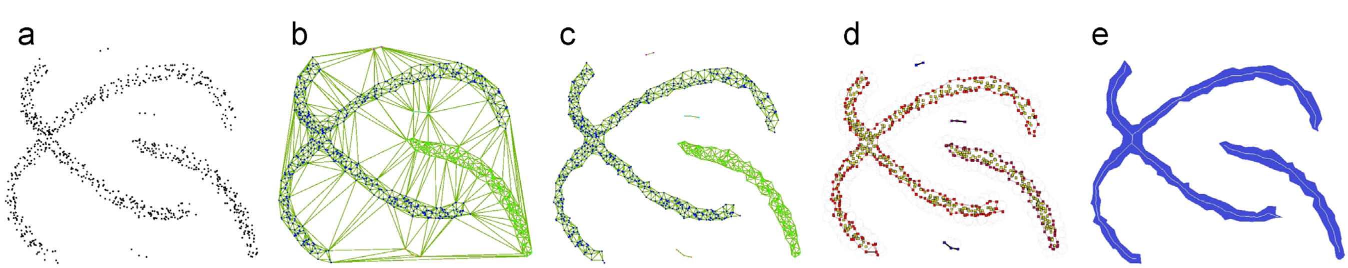

Fig. 2. A Delauney triangulation based method for finding curve components and removing outliers. The data points lie in a bounding box of size 650 by 450. (a) Data points. (b) Delauney triangulation of data points. (c) Mesh after removing long edges e where $length(e) > 23$. (d) The α-shape of data points. (e) Image generated by filling remaining triangles after deleting outliers. An skeleton is also shown which is obtained by applying image thinning algorithm and removing spurious tails. Note two vertices in the skeleton which are very close to each other will be merged into a single vertex.