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Parametric surface and properties defined on parallelogrammic domain

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Abstract

Similar to the essential components of many mechanical systems, the geometrical properties of the teeth of spiral bevel gears greatly influence the kinematic and dynamic behaviors of mechanical systems. Logarithmic spiral bevel gears show a unique advantage in transmission due to their constant spiral angle property. However, a mathematical model suitable for accurate digital modeling, differential geometrical characteristics, and related contact analysis methods for tooth surfaces have not been deeply investigated, since such gears are not convenient in traditional cutting manufacturing in the gear industry. Accurate mathematical modeling of the tooth surface geometry for logarithmic spiral bevel gears is developed in this study, based on the basic gearing kinematics and spherical involute geometry along with the tangent planes geometry; actually, the tooth surface is a parametric surface defined on a parallelogrammic domain. Equivalence proof of the tooth surface geometry is then given in order to greatly simplify the mathematical model. As major factors affecting the lubrication, surface fatigue, contact stress, wear, and manufacturability of gear teeth, the differential geometrical characteristics of the tooth surface are summarized using classical fundamental forms. By using the geometrical properties mentioned, manufacturability (and its limitation in logarithmic spiral bevel gears) is analyzed using precision forging and multi-axis freeform milling, rather than classical cradle-type machine tool based milling or hobbing. Geometry and manufacturability analysis results show that logarithmic spiral gears have many application advantages, but many urgent issues such as contact tooth analysis for precision plastic forming and multi-axis freeform milling also need to be solved in a further study.

Keywords: Spiral bevel gear; Mathematical modeling; Parametric surface; Geometrical characteristics; Manufacturability

1. Introduction

Parametric surfaces in computer aided geometric design (CAGD) are commonly defined on a triangular, rectangular or N-sided domain. The most important surface, the non-uniform rational B-spline (NURBS) surface, which is defined on a rectangular domain, is mainly used to describe the shape of industrial products. However, due to its intrinsic properties, the NURBS surface cannot accurately depict a class of kinematic or dynamic shape, such as the tooth surfaces of spiral bevel gears.

Spiral bevel gears, the teeth of which are curved and angled away from the shaft centerline, are widely used in the power transmission of intersection axes. Unlike spur and helical gears in which teeth are generated from a cylinder blank, in spiral bevel gears, teeth are generated on a conical surface, which allows the teeth to come into contact with each other gradually. Since these gears provide excellent smoothness and load capacity, they are one of the most essential components in modern mechanical engineering. Theoretically, the tooth surfaces of spiral bevel gears are spherical involute surfaces [1]; actually, the tooth flank geometry almost completely depends on the related cutting processes. More precisely, spiral bevel gears are manufactured using cradle-type milling or hobbing machine tools; their geometrical and functional properties are thus determined by the kinematic and dynamic characteristics of different machine tools. This is why standardized spiral bevel gears are not manufactured. Park and Lee [2] utilized the spherical involute tooth profile to standardize bevel gear systems and explained the geometric characteristics and kinematic behavior of the standardized bevel gears.

Based on the milling or hobbing process, several practical approaches have been taken [3-6] to design the tooth surface of a spiral bevel gear using NURBS. Since the tooth surface is constructed from actual tooth surface sampling points [3, 4] or machining simulation points [5, 6], in the final digital model, the parametric feature information such as spiral angle, nominal pressure angle, module, etc. is completely lost. The NURBS based approach cannot be conveniently used for the parametric modeling of a spiral bevel gear.

Computer numerical control (CNC) cradle-type machine

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tools have made it possible to perform nonlinear correction motions for the pinion and gear tooth surface cutting. Thus, better tooth contact quality should be achieved by using the optimal settings of machine tools according to the tooth contact analysis (TCA) method [7-12]. Litvin et al. [8, 9] proposed a local synthesis of spiral bevel gears with localized bearing contact and the predesigned parabolic function of a controlled level for transmission errors. The pinion tooth surface is generated by roll modification and cutting ratio variation in the process. Cao et al. [10] developed a function-oriented active tooth surface design methodology to incorporate transmission errors and the contact path in the engagement process of the spiral bevel gears. Favorable shape could therefore be controlled directly before manufacturing with cradle-type machine tools. Tang et al. [11] considered the kinematical errors of machine tools and the installation errors of the gear pairs in TCA. In their proposed error tooth contact analysis (ETCA) method, more processing parameters should be recommended than in TCA in spiral bevel gears cutting. However, because tooth surface quality is very sensitive to the dynamic errors of cradle-type machine tools, cutting parameters adjustment for machine tools is time-consuming tedious work and is unavailable in most cases. Furthermore, due to the limited cutting processes, it is known that the spiral angle is not constant along the spiral bevel gear tooth. Consequently, Huston and Coy [13] believed that the inconstant spiral angle adversely modifies the tooth surface characteristics, which in turn greatly affects the load distribution, contact stress, and erratic kinematics, while inducing vibrations for the spiral bevel gears. In other words, an inconstant spiral angle cannot insure uniform kinematics and dynamics along the gear tooth with the mating gear.

The logarithmic spiral (also known as the equiangular spiral or growth curve), which commonly appears in nature, was first introduced into spiral bevel gear transmission for tooth surface description by Huston and Coy [13]. The logarithmic spiral bevel gear is considered to be an ideal spiral bevel gear due to its constant spiral angle properties. However, it is not convenient to manufacture such gears in the modern gear industry. Thus, a mathematical model suitable for accurate digital modeling and the differential geometrical characteristics of the tooth surface have not yet been thoroughly investigated. Tsai and Chin [14] applied the logarithmic spiral in bevel gear systems. They provided a relative complex mathematical description of the spiral tooth surface by solving equation systems. Hence, the surface representation does not have intuitive geometric meaning and is unsuitable for manufacturability analysis. Based on intuitive space geometry and kinematic theory, Li et al. [15] derived the spatial equation of the tooth surface. However, the form of the derived equation is difficult to understand without the help of professional tools such as CAD or Matlab platform. Recently, precision plastic forming processes, such as forging and cold extrusion have made it possible to mass produce small module spiral bevel gears that are widely used in the automobile industry [2, 16]. Meanwhile, general multi-axis CNC milling machine tools (rather than the special cradle-type machine tools), have also made it possible to manufacture high precision large module gears in small batches for the shipbuilding industry [2, 17].

This paper is divided into five sections. In Section 1, the most intuitive mathematical model for the tooth surface of the logarithmic spiral bevel gear is proposed. Section 2 discusses the unified parametric surface definition on the parallelogrammic domain for different kinematic styles. The differential geometrical characteristics which will be useful for understanding tooth geometry and its manufacturability are then explained in Section 3. The logarithmic spiral bevel gear manufacturability with precision plastic forming processes and multi-axis freeform surface milling processes is then analyzed in Section 4. Section 5 then presents a discussion of the application advantages and the many urgent issues that still need to be solved. Our main contributions are the unified parametric representation of the logarithmic spiral tooth surface, and the manufacturability analysis of the logarithmic spiral bevel gear with derived surface intuitive properties.

2. Tooth surface geometry

We focus on the most important factors that influence the configuration of an accurate mathematical model of the spiral bevel gear tooth surface. In order to meet a constant spiral angle transmission condition, we consider the logarithmic spiral curve in the tooth surface geometry.

2.1 Spherical involute

The spherical involute geometry is well described by Shunmugam et al. [18]. In addition, other works on the spherical involute geometry can be referred to [2, 14]. For ease of understanding Section 2.3, we illustrate the spherical involute geometry in this section as presented in [18].

The basic kinematic characteristics of a bevel gear pair may be described using the pitch cones and base cones. Un-

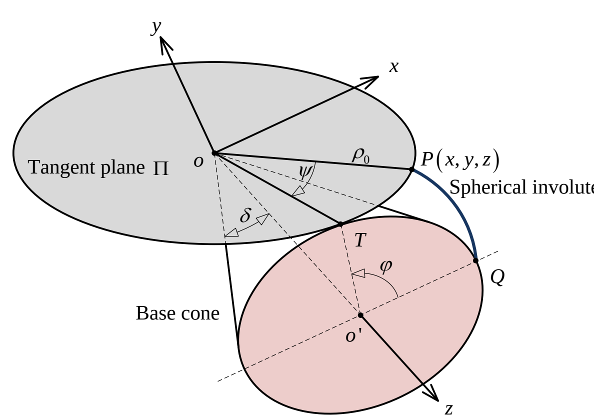

Figure 1. Spherical involute geometry.