Available online at www.sciencedirect.com
ScienceDirect
Journal of Computational Design and Engineering 2 (2015) 129–136
www.elsevier.com/locate/jcde

Variational surface design under normal field guidance

Weidong Wu, Xunnian Yang*

Department of Mathematics, Zhejiang University, Hangzhou, China

Received 25 November 2014; received in revised form 4 December 2014; accepted 17 December 2014
Available online 9 March 2015

Abstract

This paper proposes a novel method for shape design of a Bézier surface with given boundary curves. The surface is defined as the minimizer of an extended membrane functional or an extended thin plate functional under the guidance of a specified normal field together with an initial prescribed surface. For given boundary curves and the guiding normal field, the free coefficients of a Bézier surface are obtained by solving a linear system. Unlike previous PDE based surface modeling techniques which construct surfaces just from boundaries, our proposed method can be used to generate smooth and fair surfaces that even follow a specified normal field. Several interesting examples are given to demonstrate the applications of the proposed method in geometric modeling.

Keywords: Membrane energy functional; Thin plate energy functional; Normal vector field; Geometric modeling

1. Introduction

Functional optimization technique is a general approach to fair surface design [1–4]. Moreton and Séquin [1] proposed a method for the creation of smoothly connected surfaces of any genus or topological type. Welch and Witkin [2] achieved fair surfaces by functional optimization of the surface shape. The users were able to control the surface shape by attaching points and curves to the surface. Fair surface design can also be formulated as solving partial differential equations (PDEs) subject to geometric or physical constraints [5–12]. In literature [6], a system was proposed for global and local deformations of PDE-based surface models subject to physical constraints. At the same time, the system also computed the B-spline finite element approximation of the PDE surface and allowed users to interactively manipulate the surface.

The tensor product Bézier surfaces, B-spline surfaces and NURBS surfaces are widely used in surface shape design. By employing the technique of control points, these surfaces can be designed interactively. Generally, the parametric surfaces can be deformed by searching the control points and weights subject to the geometric constraints [13–19]. Hu et al. [20] proposed two methods for modifying the shape of NURBS surfaces with geometric constraints, such as points, normal vectors at selected points, and pre-constructed curves. Both methods are dedicated to changing the control points and weights of an initial surface. Sauvage et al. [21] addressed the deformation of B-spline surfaces while constraining the volume enclosed by the surface. Pusch et al. [22] proposed an algorithm for locally deforming either a parametric surface or a hierarchical subdivision surface to match a set of positional and energy minimizing constraints.

Among all functionals for fair surface design, Dirichlet functional [23] and bi-harmonic functional [24] are popular for generating smooth and fair surfaces that interpolate given boundary curves. However, surfaces generated by these two functionals have few degrees of freedoms for shape adjustment, and they cannot represent even cylinder or cone like surfaces which are widely used in CAD. The geometric PDE method can generate typical surfaces for shape modeling [25] and surface restoration [26]. But, these equations are hard to have analytic solutions due to high nonlinearity. For applications like transition surface design or hole filling, the interpolating surfaces may have salient features which should

*Corresponding author.
E-mail addresses: wuweidong.happy@163.com (W. Wu), yxn@zju.edu.cn (X. Yang).
Peer review under responsibility of Society of CAD/CAM Engineers.

http://dx.doi.org/10.1016/j.jcde.2015.03.001

be controlled by additional parameters. This motivates us developing new functionals for fair surface design that have enough degrees of freedoms for shape adjustment as well as explicit solutions.

We propose to design Bézier surfaces with given borders by minimizing an extended membrane energy or an extended thin plate energy. Besides being as fair as possible, the resulting surface also fits to a prescribed normal vector field and an initial prescribed surface. Two shape parameters λ and γ are introduced to balance the effects of normal field and the initial surface. If λ and γ are chosen zero, the energy functional will degenerate to the Dirichlet functional or the bi-harmonic functional. The algorithm is easy to implement and the free control points of the surface are obtained by solving a linear system. For the convenience of adjusting the specified normal vector field, we can discretize the functional on a grid of parametric points and specify a discrete normal vector field. We have applied the proposed method for surface editing, hole filling and transition surface modeling.

The paper is organized as follows. In Section 2, an extended membrane energy functional and an extended thin plate energy functional are introduced. In Section 3, we propose explicit formulae for variational surface design under the guidance of normal vector field with given borders. Variational surface design under the guidance of discrete normal field is given in Section 4. In Section 5 we present several interesting examples. Section 6 concludes the paper.

2. Extended energy functionals for shape optimization

In this section, we propose two new energy functionals for Bézier surface shape design. These functionals are defined on the space of Bézier patches $\mathbf{R}:\Omega\rightarrow\Re^3$. Assuming $\mathbf{R}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_i^n(u)B_j^m(v)\mathbf{P}_{ij}$ is the Bézier surface to be designed, $\mathbf{N}(u,v)=\sum_{i=0}^{n_1}\sum_{j=0}^{m_1}B_i^{n_1}(u)B_j^{m_1}(v)\mathbf{N}_{ij}$ is the prescribed normal function and $\mathbf{S}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_i^n(u)B_j^m(v)\hat{\mathbf{P}}_{ij}$ is the given Bézier surface. We would like to find a fair surface $\mathbf{R}(u,v)$ that lies close to the given surface $\mathbf{S}(u,v)$ and fits well to the known normal field $\mathbf{N}(u,v)$.

First, we extend the membrane energy functional by allowing the surface to follow the shape of a given normal field and an initial surface. The extended membrane energy functional is given by

$$E_1(\mathbf{R})=\frac{1}{2}\int_\Omega\left\{\mathbf{R}_u^2+\mathbf{R}_v^2+\lambda\left[(\mathbf{R}_u\cdot\mathbf{N})^2+(\mathbf{R}_v\cdot\mathbf{N})^2\right]+\gamma(\mathbf{R}-\mathbf{S})^2\right\}du\,dv, \tag{1}$$

where $\mathbf{R}_u,\mathbf{R}_v$ are the partial derivatives of $\mathbf{R}$, and $\lambda(\geq 0)$, $\gamma(\geq 0)$ are the coefficients chosen by users. If $\lambda>0$, the resulting surface fits to the prescribed normals. If $\gamma>0$, the resulting surface follows the shape of the initial given surface also. The corresponding Euler–Lagrange equation of the functional is

$$\begin{aligned}0=&(\mathbf{I}+\lambda\mathbf{N}\mathbf{N}^t)(\mathbf{R}_{uu}+\mathbf{R}_{vv})\\&+\lambda[(\mathbf{N}\mathbf{N}_u^t+\mathbf{N}_u\mathbf{N}^t)\mathbf{R}_u+(\mathbf{N}\mathbf{N}_v^t+\mathbf{N}_v\mathbf{N}^t)\mathbf{R}_v]\\&-\gamma(\mathbf{R}-\mathbf{S}),\end{aligned}$$

where $\mathbf{I}$ is the identity matrix and t represents the transpose of a column vector. If both the coefficients λ and γ vanish, the functional reduces to the classical Dirichlet functional, and the corresponding Euler–Lagrange equation becomes the classical Laplacian equation.

Second, we extend the thin plate functional by using the prescribed normal field. In addition to interpolating the given boundary curves, the thin plate energy can be used to optimize surfaces that interpolate given tangent planes at the boundaries. In a similar fashion to the functional (1), the extended thin plate functional is defined as

$$\begin{aligned}E_2(\mathbf{R})=\frac{1}{2}\int_\Omega\{&\mathbf{R}_{uu}^2+2\mathbf{R}_{uv}^2+\mathbf{R}_{vv}^2\\&+\lambda[(\mathbf{R}_u\cdot\mathbf{N})^2+(\mathbf{R}_v\cdot\mathbf{N})^2]\\&+\gamma(\mathbf{R}-\mathbf{S})^2\}\,du\,dv,\end{aligned}\tag{2}$$

where $\mathbf{R}_{uu},\mathbf{R}_{uv},\mathbf{R}_{vv}$ are the second derivatives of $\mathbf{R}$. The corresponding Euler–Lagrange equation for this functional is

$$\begin{aligned}0=&\mathbf{R}_{uuuu}+2\mathbf{R}_{uuvv}+\mathbf{R}_{vvvv}-\lambda\mathbf{N}\mathbf{N}^t\mathbf{R}_{uu}-\lambda\mathbf{N}\mathbf{N}^t\mathbf{R}_{vv}\\&-\lambda[(\mathbf{N}\mathbf{N}_u^t+\mathbf{N}_u\mathbf{N}^t)\mathbf{R}_u+(\mathbf{N}\mathbf{N}_v^t+\mathbf{N}_v\mathbf{N}^t)\mathbf{R}_v]+\gamma(\mathbf{R}-\mathbf{S}).\end{aligned}$$

When the coefficients λ, γ vanish, the functional degenerates to the thin plate functional. The corresponding Euler–Lagrange equation becomes the biharmonic equation.

3. Variational surface design with given borders

Though the minimizer of functional in Eq. (1) or in Eq. (2) can be characterized by the Euler–Lagrange equation, practical applications such as filling holes or designing transition surfaces usually need to solve fair surfaces with known boundaries. In the following we minimize the functional (1) or (2) by assuming the boundary curves or the boundary control points of a Bézier surface are already given. As the integrals in Eq. (1) or (2) can be computed explicitly, the free control points of the Bézier surface will be finally obtained by solving a linear system.

3.1. Surface modeling by minimizing the extended membrane energy

When we model a surface by minimizing the extended membrane energy functional with the given boundary curves, the minimization problem can be converted to solving the following system of equations.

$$\frac{\partial E_1}{\partial \mathbf{P}_{ij}}=0\quad(i=1,\ldots,n-1;j=1,\ldots,m-1).$$

Since the energy functional $E_1(\mathbf{R})$ is a quadratic functional in terms of the unknown control points, the mentioned equations form a linear system.

Let

$$\begin{aligned}\mathbf{C}_{kl}^{ij}=n^2\int_\Omega\{&[B_{i-1}^{n-1}(u)-B_i^{n-1}(u)]B_j^m(v)B_k^{n-1}(u)B_l^m(v)\\&(\mathbf{I}+\lambda\mathbf{N}\mathbf{N}^t)\}\,du\,dv,\quad k=0,1,\ldots,n-1,l=0,1,\ldots,m\end{aligned}$$