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Multicriteria shape design of an aerosol can

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Abstract

One of the current challenges in the domain of the multicriteria shape optimization is to reduce the calculation time required by conventional methods. The high computational cost is due to the high number of simulation or function calls required by these methods. Recently, several studies have been led to overcome this problem by integrating a metamodel in the overall optimization loop. In this paper, we perform a coupling between the Normal Boundary Intersection – NBI – algorithm with Radial Basis Function – RBF – metamodel in order to have a simple tool with a reasonable calculation time to solve multicriteria optimization problems. First, we apply our approach to academic test cases. Then, we validate our method against an industrial case, namely, shape optimization of the bottom of an aerosol can undergoing nonlinear elasto-plastic deformation. Then, in order to select solutions among the Pareto efficient ones, we use the same surrogate approach to implement a method to compute Nash and Kalai–Smorodinsky equilibria.

Keywords: Multicriteria optimization problem; Normal boundary intersection; Radial basis function metamodel; Nash equilibria; Kalai-Smorodinsky equilibria

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1. Introduction

Structural multidisciplinary shape optimization – MDO – is known to demand costly computational resources, notably when one seeks to identify the Pareto front, one of the most relevant MDO tools. To overcome this obstacle, it is classical to couple methods for the Pareto capture with metamodels aimed at cheap costs evaluation [1–5]. There are two possible couplings between methods to identify the set of Pareto optimal solutions, and metamodels: The first idea is to lead optimization with the dedicated algorithms (NBI or others) and use an updated metamodel for a certain number of evaluations until finding the solutions (strong coupling). The second idea is to lead optimization with the metamodel and only do the exact calculations of the metamodel-obtained solutions (weak coupling).

In our work, the Normal Boundary Intersection (NBI) method [6–8] and the radial basis function (RBF) metamodel [10–14] are used to build our algorithm (NBI RBF) using a weak coupling. The implemented algorithm is validated against mathematical test-cases, and then used to perform a multicriteria shape optimization of structures which undergo highly nonlinear deformations. We compare the results obtained for different a priori discretizations of the Pareto fronts. We also address the problem of selecting solutions among the Pareto optimal ones by using a Nash game approach [22–26] and a Kalai–Smorodinsky one [19–21].

2. Methodology

In this section, we present the methodology and background used throughout the paper.

2.1. Multicriteria optimization and Pareto optimality

A multicriteria optimization problem is given as follows:

$$\min_x \quad F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^T \quad m \geq 2$$

$$\text{subject to} \quad (D)\begin{cases} g_j(x) \geq 0, & j = 1, \ldots, J \\ h_k(x) = 0, & k = 1, \ldots, K \\ x^{lower} \leq x \leq x^{upper} \end{cases} \tag{1}$$

where m, J and K are the total numbers of the objective functions, the inequality (g_j) and equality constraints (h_k), respectively.

The Pareto front is defined as the set of non-dominated designs, in the objective space. A design point, $x^* \in D$ is non-dominated if there is no other point, $x^* \in D$, such that

$$f_i(x) < f_i(x^*), \quad i = 1, \ldots, m$$

with strict inequality for at least one index.

2.2. Normal boundary intersection

Normal boundary intersection method NBI is a solution methodology developed by Das and Dennis (1996) for the approximation of Pareto surfaces [9]. The method is based on the intersection of the so-called CHIM's (convex hull of individual minima) normal and the objective space boundary.

We summarize it as follows:

Let x_i^* be the respective global minimizers of $f_i(x)$, $i=1,\ldots,m$ over $x \in (D)$.

Let $F_i^* = F(x_i^*)$, $i=1,\ldots,m$.

Let $F^* = [f_1(x_1^*), f_2(x_2^*), \ldots, f_m(x_m^*)]^T$.

Let $\beta \in R^m$ a weight vector.

Let ϕ be the $m \times m$ matrix whose ith column is $F(x_i^*) - F^*$ known as the pay-off matrix.

Then the set of points in R^m that are convex combinations of $F(x_i^*) - F^*$ is referred to as the CHIM, i.e., $CHIM = \{\phi\beta, \beta \in R^m \text{ with } \sum_{i=1}^m \beta_i = 1, \beta_i \geq 0\}$. The set of attainable objective vectors $\{F(x) : x \in (D)\}$ is denoted by $\boldsymbol{F}$ and is usually referred to as the objective space. Let us denote the boundary of $\boldsymbol{F}$ by $\partial \boldsymbol{F}$.

Let $\mathbf{n}$ denote the unit normal to the CHIM simplex pointing towards the origin defined as

$$\mathbf{n} = \{-\phi e, \; e \in R^m \text{ with } e = \{1, 1, 1, \ldots, 1\}\}$$

NBI method determines the portion of $\partial \boldsymbol{F}$ which contains the Pareto optimal points. The principal idea behind this approach is that the intersection point between the boundary $\partial \boldsymbol{F}$ and the normal $\mathbf{n}$ pointing towards the origin emanating from any point in the CHIM is a point on the portion of $\partial \boldsymbol{F}$ containing the efficient points. This point is guaranteed to be a Pareto optimal point if the trade-off surface in the objective space is convex. This is the algebraic idea behind NBI's approach, and Das and Dennis have shown that this approach can be written mathematically and also that the point of intersection of the normal and the boundary of $\boldsymbol{F}$ closest to the origin is the global solution of the following single problem:

$$\max_{x,t} \quad t$$

$$\text{subject to} \quad (D_{NBI})\begin{cases} \phi \cdot \beta + t \cdot \mathbf{n} = F(x) - F^* \\ g_j(X) \geq 0, & j = 1, \ldots, J \\ h_k(X) = 0, & k = 1, \ldots, K \\ x^{lower} \leq x \leq x^{upper} \end{cases} \tag{2}$$

where $t \in R$ is a dummy parameter, the sub-problem (Eq. (2)) is repeated for a number (N) of different weight of β, and only one solution is obtained for each weight, and finally, the NBI method gives us N solutions for the initial problem (Eq. (1)).