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# Quadrilateral mesh fitting that preserves sharp features based on multi-normals for Laplacian energy

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**Abstract**

Because the cost of performance testing using actual products is expensive, manufacturers use lower-cost computer-aided design simulations for this function. In this paper, we propose using hexahedral meshes, which are more accurate than tetrahedral meshes, for finite element analysis. We propose automatic hexahedral mesh generation with sharp features to precisely represent the corresponding features of a target shape. Our hexahedral mesh is generated using a voxel-based algorithm. In our previous works, we fit the surface of the voxels to the target surface using Laplacian energy minimization. We used normal vectors in the fitting to preserve sharp features. However, this method could not represent concave sharp features precisely. In this proposal, we improve our previous Laplacian energy minimization by adding a term that depends on multi-normal vectors instead of using normal vectors. Furthermore, we accentuate a convex/concave surface subset to represent concave sharp features.

*Keywords*: CAD model; Hexahedral mesh; Sharp feature; Fitting algorithm; Multi-normalvectors

## 1. Introduction

In manufacturing, the cost of computer simulations is lower than testing actual prototypes. Thus, most manufacturers run simulations, which require volume meshes. Up until a decade ago, the simulation process started from surface meshes that were made using computer aided design (CAD) software. However, there were differences in the shape between the actual products and the CAD model, a result of manufacturing factors such as springback in production presses. Even if manufacturers used metal dies with the same shapes as in the CAD models, the parts obtained from the processes such as press working do not have the same shape as the CAD model. Thus, the actual products are different from the CAD model. For realistic simulations, the CAD model must be identical to the products. Today, the simulation process starts from point clouds scanned from the actual products. This process is called reverse engineering.

Tetrahedral/hexahedral meshes generated from such point clouds directly affect the results in finite element method (FEM) analysis. Hexahedral meshes are important because they are superior to tetrahedral meshes (see Figure 1(a)) in terms of accurate analysis. Thus, in this paper, we use hexahedral volume meshes (see Figure 1(b)) whose elements are only hexahedral cells (called all-hexahedral meshes) and consider their surface.

In the structural analysis between two objects, the peak stress occurs near or around the contact regions. Such regions are often sharp features. A sharp feature is typically a cusp part (such as an edge or point) of an object. Thus, to obtain accurate simulation results, the surface mesh must represent sharp features.

In this paper, we consider a voxel-based hexahedral mesh generation algorithm [1, 2]. In addition, the volume mesh for the FEM must satisfy the constraint that all Jacobians are positive. The Jacobian is a triple scalar product $a \cdot (b \times c)$, where $a$, $b$ and $c$ are vectors (edges) adjacent to the corner vertex of a cell. Our ultimate goal is automatic hexahedral mesh generation without negative Jacobians. In this paper, we discuss the quadrilateral surface of a hexahedral mesh (The algorithm we investigated in previous studies is voxel-based [1, 2]). Thus, our inputs are the target surface mesh and the quadrilateral mesh that is the surface of the voxel mesh.

Generically, we can classify the methods of boundary-fitted hexahedral meshing as voxel-based [3, 4], advancing front [5-7], whisker waving [8], cycle elimination [9], medial axis-based [10], and sweep/mapped methods [11, 12], although other types exist. Hexahedral mesh generation algorithms can be fully or semi-automatic, but there is no scheme that guarantees all Jacobians will be positive. On the other

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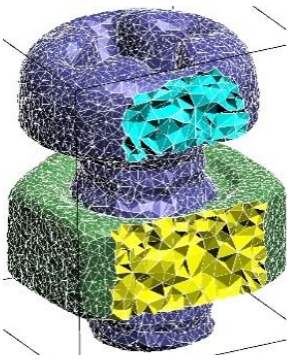


(a)

(b)

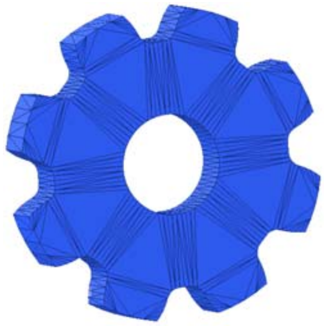
Figure 1. Tetrahedral/hexahedral meshes: (a) tetrahedral mesh, (b) hexahedral mesh.

hand, there is a scheme that guarantees that positive Jacobians does exist for tetrahedral meshes. Before considering how to achieve positive Jacobians, we generate quadrilateral surfaces of the all-hexahedral meshes that represent the target surfaces. In this paper, we propose an automatic fitting algorithm with sharp features based on previous works [1, 2, 13].

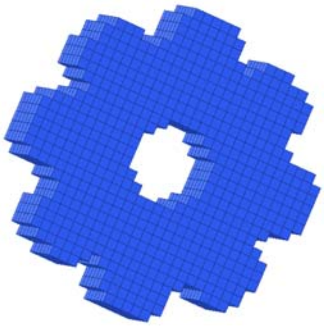
## 2. Previous hexahedral meshing

First, we summarize the previous hexahedral mesh generation algorithm [1, 2, 13]. The underlying algorithm can be broken down as follows.

1. Input target surface mesh (see Figure 2(a) [14]).
2. Generate voxels to wrap around the target surface using Polymender [15] (see Figure 2(b)).
3. Extract the boundary surface of voxels.
4. Fit the boundary surface of voxels (see the fitted surfaces in Figure 2(c) [1, 2] and Figure 2(d) [13]).
5. Determine the positions of the inner vertices.
6. Apply post-processing.
7. Output hexahedral mesh.



(a)



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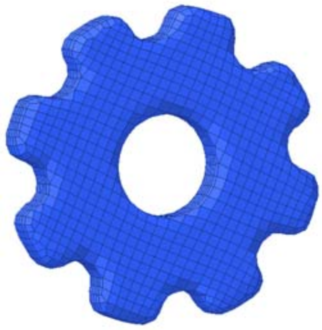

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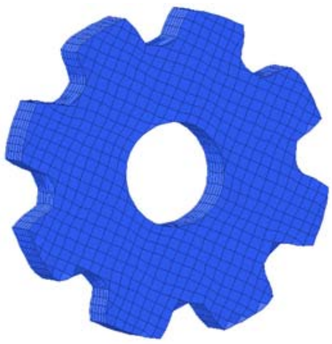

(d)

Figure 2. Gear: (a) target surface (triangle mesh), (b) voxels of gear, (c) quadrilateral mesh without sharp features, (d) with sharp features.