Graphical Models 81 (2015) 1–17

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

A new optimization approach for mass-spring models parameterization[☆]

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ARTICLE INFO

Article history:
Received 2 January 2015
Revised 20 May 2015
Accepted 15 July 2015
Available online 29 July 2015

Keywords:
Mass-spring model
Mass-spring parameterization
Deformable object models
Physically based modeling
Finite element method
Elasticity theory

ABSTRACT

Mass-spring models (MSM) are frequently used to model deformable objects for computer graphics applications due to their simplicity and computational efficiency. However, the model parameters are not related to the constitutive laws of elastic material in an obvious way. The MSM parameters computation from a model based on continuum mechanics is a possibility to address this problem. Therefore, in this paper we propose a new method to derive MSM parameters using a data-driven strategy with a new objective function based on the model acceleration so that the MSM and the reference model behave similarly. The proposed methodology does not depend on reference model, mesh topology or static equilibrium configuration. We validate the methodology for deriving MSM systems using finite element method (FEM) and MSM itself as reference models. The obtained results are compared with related works. We also discuss its robustness against different discretizations and material properties.

1. Introduction

In the last decades, a wide variety of physically based models has been developed by the computer graphics community to address the challenge of simulating natural elements and deformable materials. For the latter, constitutive laws are used for the computation of the symmetric internal stress tensor and a conservation law gives the final partial differential equation (PDE) that governs the dynamics of the material [1,2]. Continuous systems have infinite degrees of freedom which make its description difficult for both the geometric and dynamic aspects. In mathematical terms, we are dealing with infinite basis functions, maybe uncountable. One possibility to simplify the problem is to consider finite dimensional representation with enough flexibility in order to represent the solution with the desired precision. In the context of mechanical systems the finite element method (FEM) is the traditional way to perform this task. When simulating a deformable body, the 3D/2D object's geometry is usually represented by mesh based methods that offer the support for FEM-based techniques [3].

Other possibility for elastic objects simulation is to apply discrete models, based on mass-spring systems which are known, among other nomenclatures, as mass-spring models (MSM). In this case, the object's geometry is represented by a mesh and its nodes are treated like mass points while each edge acts like a spring connecting two adjacent nodes. MSM are simple to implement and can be faster than the continuous ones, and so, more suitable for real time applications [4]. Therefore, MSM techniques have been used to model deformable objects [2], for woven cloth simulation simulation [4] and soft organic tissues, like muscles, face or abdomen in virtual surgery applications [5–9].

However, the main limitation of the MSM is the difficulty of designing them to represent the mechanical behavior of

☆ This paper has been recommended for acceptance by Ladislav Kavan and Peter Lindstrom.

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http://dx.doi.org/10.1016/j.gmod.2015.07.001
1524-0703/

deformable bodies with enough accuracy [10]. The relation between mass-spring models and the continuum elasticity theory was examined in references [10–12]. The conclusion is that methods that are based on the continuum mechanics are, in general, more realistic than their discrete counterparts. This happens because mechanical systems depend on their macroscopic parameters (Young's module and Poisson's ratio) and constitutive equations that characterize the nature of the materials that make up the bodies [3]. However, there is no general physically based or systematic method in the literature to determine the mesh topology or MSM parameters from known constitutive behavior [13].

When comparing mass-spring and continuous models the following questions arise: (1) Which set of elastic material properties can be accurately simulated by a particular spring mesh model? (2) How to derive a mass-spring system from continuum mechanics [14]? (3) How to specify system parameters (masses, spring constants, mesh topology) in order to match physical requirements [11]?

The questions (1) and (3) have been more or less addressed by works that adapt MSM to describe different elastic behaviors such as anisotropy, heterogeneity, nonlinearity and also incompressibility (see [15] and references therein).

In this paper, we address the question (2) with focus on the simulation of deformable objects in real-time for application in virtual environments. We propose a new method to derive mass-spring systems using a data-driven strategy which is roughly composed by four stages: (a) simulate the deformable object using a reference model and keep the position and velocities of the particles; (b) solve an optimization problem based on the acceleration of the reference and MSM models in order to compute the stiffness parameters; (c) calculate particles masses following [16,17]; (d) compute the damping parameters using a technique based on [18].

The obtained solution gives parameters (damping, stiffness coefficients and particles masses) that allow the MSM and the reference model behaves similarly. In order to avoid computational complexity to compute the solution we develop an algorithm to calculate an approximation of the global solution which identifies the dominant term in the objective function and performs its minimization with respect to the target parameters. We offer a theoretical analysis to justify our proposal as well as some implementation details. We validate the proposed method in the context of 2D and 3D isoparametric FEM models. The obtained results show the efficiency of our methodology when compared with related ones. Also, we analyze the robustness of our algorithm against different discretizations, optimization problem setups and material properties.

The paper is organized as follows. Section 2 describes the related literature. Next, in Section 3, we offer the necessary background in FEM and MSM models. The proposed approach is described on Section 4. The experimental results are presented and discussed on Section 5. Finally, we present in Section 6 the conclusions and future works.

2. Related works

Two categories of methods can be identified in the estimation of the parameters for MSM in order to guarantee a realistic behavior: data-driven and model driven. The first category is composed by methods that use a minimization procedure to find the model that shows the closest behavior to that of the observed (or simulated) deformable object. To implement such solution we must specify some properties and/or constraints for the MSM and then to seek for the other ones by optimizing an objective function that measures the similarity between the configurations of both the MSM and the reference model. In these cases, it is common to use genetic algorithms [19] and simulated annealing [20]. All these methods share the same basic principle: applying random values to different springs properties and correct the ones that induce the greatest error in order to minimize the discrepancies. These methods are well suited for solving complex problems involving non-linearity and can handle with discrete properties of the MSM configuration, like the mesh topology [19]. The main disadvantage of the use of these methods is the need for long computation times.

The second class is composed by those methods that try to obtain the values of mass, stiffness and damping ratio that reproduce a known property of the reference model through theoretical considerations [6]. These methods seek to derive parameters starting from some analytical knowledge of the material or model, such as the FEM. For instance, we can seek for a linearized MSM model that produces elements with a stiffness matrix similar to that from linear FEM. This reasoning strategy based on the FEM formulation was started by Van Gelder [11] who initially derived an MSM, with topology given by a triangular mesh, equivalent to the FEM in the context of linear elasticity. Specifically, it derives a formula for computing the spring stiffness coefficient of an edge according to the geometry of the triangles incident upon that edge as well as material properties (Young's modulus). However, this approach did not show positive results and the conclusion was that, in general, there is no possible solution that matches FEM and MSM stiffness matrices.

Some years later, Lloyd et al. [10] demonstrated that there is a particular case where both matrices are equal. This particular case occurs when using equilateral triangle finite element and Poisson's ratio equal to 1/3. The approach results in explicit formulas for the MSM stiffness coefficients for triangle, rectangle, and tetrahedron meshes. An extension of this work is found in [17] which presents formulas to derive the dynamic MSM parameters (mass and damping) as well. Also, it was demonstrated in [15] that Van Gelder's approach is restricted to null Poisson's ratio. In this reference it is supposed a linear elastic, isotropic and homogeneous materials and spring coefficients are determined to correctly simulate shear, elongation (tensile) for these mechanical systems. Firstly, the method computes the associated Lagrangian which depends on variables related to the material response to the shearing/elongation stress. Next, expressions for the Lagrangian extremum are computed to compose a system of equations together with the measured mechanical characteristics definitions. The idea is to build a set of equations whose solutions give the spring coefficients as function of the mechanical characteristics.

On the other hand, MSM models can be derived from a continuum approach by interpreting local expressions of energy or force terms computed by finite difference or FEM formulations. In this way, in [12] it is considered an isotropic membrane represented by a triangular mesh and modeled