Graphical Models 80 (2015) 31–40

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

A fast algebraic non-penetration filter for continuous collision detection

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ARTICLE INFO

Article history:
Received 17 March 2015
Revised 14 May 2015
Accepted 7 June 2015
Available online 12 June 2015

MSC:
00-01
99-00

Keywords:
Continuous collision detection
Non-penetration filter
Cubic equation
Deformable objects

ABSTRACT

Continuous collision detection is a key technique to meet non-penetration requirements in many applications. Even though it is possible to perform efficient culling operations in the broad stage of a continuous collision detection algorithm, such as bounding volume hierarchies, a huge number of potentially colliding triangles still survive and go to the succeeding narrow stage. This heavily burdens the elementary collision tests in a collision detection algorithm and affects the performance of the entire pipeline, especially for fast moving or deforming objects. This paper presents a low-cost filtering algorithm using algebraic analysis techniques. It can significantly reduce the number of elementary collision tests that occur in the narrow stage. We analyze the root existence during the time interval [0, 1] for a standard cubic equation defining an elementary collision test. We demonstrate the efficiency of the algebraic filter in our experiments. Cubic-solvers augmented by our filtering algorithm are able to achieve up to 99% filtering ratios and more than 10 × performance improvement against the standard cubic-solver without any filters.

1. Introduction

Enforcing non-penetration constraints between movable/deformable objects has been widely used in collision responses in physically-based simulation [1,2], motion planning in robotics [3], god-object computation in haptic rendering [4], tolerance verification in virtual prototyping [5], etc. Continuous collision detection (CCD) is one of the major techniques to maintain non-penetration constraints and to robustly handle collision responses. For movable/deformable objects, two filtering techniques have been used to accelerate the queries of continuous collision detection: dynamic bounding volume hierarchies (BVHs) in broad stage [6] and non-penetration filters [7] in narrow stage. More specifically, in broad stage, dynamic BVHs is used to efficiently localize the potentially colliding regions by traversing precomputed BVHs. In narrow stage, non-penetration filters can be used to reduce the false positives of elementary collision tests for those potentially colliding triangles. Here, false positive means that a collision test is performed, but no collision is found.

For fast moving objects and severely deformable objects, the filtering efficiency of dynamic BVHs may significantly degrade due to over-conservativeness of bounding volumes, so that a large number of potentially colliding triangles survive from the broad stage and go to the narrow stage. In the narrow stage, continuous collision detection is performed for a huge number of moving/deforming triangles, which eventually boils down to performing hundreds of thousands or even tens of millions elementary collision tests between triangle primitive features. For example, in our cloth dynamics benchmark (see Fig. 5), the elementary collision tests vary from 13 thousands to 52 millions for each frame through the

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http://dx.doi.org/10.1016/j.gmod.2015.06.001
1524-0703/

entire simulation. Given the high number of potentially colliding triangles survived from the broad stage, the use of continuous collision detection can be limited by its higher computational costs than discrete collision detection. We restrict our attention to reducing elementary collision tests in the narrow stage.

There are two types of elementary collision tests between two moving/deforming triangles: a vertex against a triangular face (face-vertex test) and an edge against another edge (edge-edge test) [8]. Continuous collision detection for a pair of deforming triangles includes 15 elementary collision tests, including six face-vertex tests and nine edge-edge tests. If none of these 15 elementary tests reports collision, the two triangles do not collide. Otherwise, the earliest time instance among these 15 tests is obtained as the time of contact of the two triangles. Given the high number of elementary collision tests, however, its false positive ratios are typically very high [7]. It reports to be as high as, or often even higher than 95%[1]. For elementary collision tests, false positives refer to the case where face-vertex or edge-edge pairs are tested, but do not actually intersect. In this paper, we derive a fast and low cost algebraic filter that can significantly reduce the false positives and eventually reduce the number of elementary collision tests in the narrow stage.

We organize the rest of this paper as follows. The related work is given in Section 2. The filtering problem, its formulation and the main ideas of our algebraic filtering algorithm are described in Section 3. Our filtering algorithm is given in Section 4, which is based on Vincent's theorem. The implementation details, experimental results and comparisons are given in Section 5. The paper is concluded in Section 6.

2. Related work

There are two main approaches of performing elementary CCD tests for moving objects: algebraic equation solvers [2,8,9] and local conservative advancement [10,11]. As for the former, Moore and Wilhelms [9] first presented a solution using fifth order algebraic equations and later Provot [8] reduced them to cubic equations. The elementary CCD test for moving triangles reduces to perform face-vertex tests (a vertex against a triangular face) and edge-edge tests (an edge against another edge). In Provot's formulation, each of them corresponds to a cubic algebraic equation (refer to Section 3). This approach has been used as a standard method in many applications such as games, virtual environment, and interactive computer graphics. As for the latter, given a closest distance between two triangle features, local conservative advancement states that the two features must travel at least this distance in order to come into contact; otherwise, the two features can move without worrying about collision. Repetitively applying conservative advancement to the two triangle features can be used for obtaining the lower bound of the time of contact between these two features. The conservative advancement technique has been used for rigid [12], articulated [6] and deformable objects [10,11].

To reduce redundant elementary collision tests, connectivity (i.e. adjacency), geometric constraints and dimension reduction have been used in [7,13–18]. More specifically, Crutis et al. [13] presented representative-triangles (R-Triangle) to eliminate duplicate elementary tests. Tang et al. [7] derived a non-penetration filter using coplanarity and convex hull properties. Tang et al. [18] reduced redundant elementary tests in 1D and 2D subspace. Continuous normal cones [15] can be used to remove false positives for regions with relatively low curvatures. An exact algorithm was presented in [19] to eliminates 99% elementary tests, but its computational cost is very high. Two recent work [20,21] handled exact CCD problem using error analysis and Bernstein sign classification. In [11], a sequence of simple feature-level filtering tests based on local conservative advancement are used to accelerate continuous collision detection for deforming triangles.

3. Elementary collision tests

3.1. Cubic equations

For a pair of moving/deforming triangles, we assume that their vertices move with constant velocity during the time step $t \in [0, 1]$ and that the triangles are linearly interpolated between their vertices at intermediate times. Continuous collision detection between two moving/deforming triangles includes a set of face-vertex tests or edge-edge tests. Given four points $\vec{x}_i(t)(i = 1, 2, 3, 4)$ and their constant velocities $\vec{v}_i(i = 1, 2, 3, 4)$, a distance function $\mathrm{d}(t)$ between a vertex $\vec{x}_4$ and a triangle $\vec{x}_1\vec{x}_2\vec{x}_3$ or between an edge $\vec{x}_1\vec{x}_2$ and another edge $\vec{x}_3\vec{x}_4$, can be defined in a unified form as follows:

$$\mathrm{d}(t) = [\vec{x}_2(t) - \vec{x}_1(t)] \times [\vec{x}_3(t) - \vec{x}_1(t)] \cdot [\vec{x}_4(t) - \vec{x}_1(t)],$$

where $t \in [0, 1]$, $\vec{v}_i = \vec{x}_i(1) - \vec{x}_i(0)$, $\vec{x}_i(t) = \vec{x}_i(0) + t\vec{v}_i$. For the sake of simplicity, we let $\vec{x}_i = \vec{x}_i(0)$. Then the above formulation can be represented as

$$\mathrm{d}(t) = [\vec{x}_{21} + t\vec{v}_{21}] \times [\vec{x}_{31} + t\vec{v}_{31}] \cdot [\vec{x}_{41} + t\vec{v}_{41}],$$

where we use the shorthands $\vec{x}_{ij}$ and $\vec{v}_{ij}$ to denote $\vec{x}_i - \vec{x}_j$ and $\vec{v}_i - \vec{v}_j$, respectively. Rearranging $\mathrm{d}(t)$ yields to the standard form of a cubic polynomial as follows:

$$\mathrm{d}(t) = a_3t^3 + a_2t^2 + a_1t + a_0$$

where

$$\begin{aligned}
a_3 &= \vec{v}_{21} \times \vec{v}_{31} \cdot \vec{v}_{41} \\
a_2 &= \vec{v}_{21} \times \vec{v}_{31} \cdot \vec{x}_{41} + (\vec{v}_{21} \times \vec{x}_{31} + \vec{x}_{21} \times \vec{v}_{31}) \cdot \vec{v}_{41} \\
a_1 &= (\vec{v}_{21} \times \vec{x}_{31} + \vec{x}_{21} \times \vec{v}_{31}) \cdot \vec{x}_{41} + \vec{x}_{21} \times \vec{x}_{31} \cdot \vec{v}_{41} \\
a_0 &= \vec{x}_{21} \times \vec{x}_{31} \cdot \vec{x}_{41}
\end{aligned}$$

$\mathrm{d}(t) = 0$ (i.e., coplanarity of the four points) is a necessary condition for collision between two triangle features (face-vertex or edge-edge), as shown in Fig. 1. A cubic equation can therefore be derived. The smallest root $t \in [0, 1]$ of the cubic $\mathrm{d}(t) = 0$ will be the time of contact for two triangle features. In practice, a collision can be reported when a time interval is smaller than a threshold during root finding. In some other algorithms, a collision occurs when the distance between the feature pairs is smaller than a threshold. Any $t \notin [0, 1]$ can be discarded. Non-existence of $t \in [0, 1]$ indicates the absence of collision between two triangle features during the time interval $[0, 1]$.

[1] Among 100 elementary tests, there are merely less than five real collisions.