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Feature-preserving surface mesh smoothing via suboptimal Delaunay triangulation ☆

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ABSTRACT

A method of triangular surface mesh smoothing is presented to improve angle quality by extending the original optimal Delaunay triangulation (ODT) to surface meshes. The mesh quality is improved by solving a quadratic optimization problem that minimizes the approximated interpolation error between a parabolic function and its piecewise linear interpolation defined on the mesh. A suboptimal problem is derived to guarantee a unique, analytic solution that is significantly faster with little loss in accuracy as compared to the optimal one. In addition to the quality-improving capability, the proposed method has been adapted to remove noise while faithfully preserving sharp features such as edges and corners of a mesh. Numerous experiments are included to demonstrate the performance of the method.

1. Introduction

Triangular surface meshes are widely used in computer graphics, industrial design and scientific computing. In computer graphics and design, people are typically interested in the smoothness (low variation in curvature) and sharp features (edges, corners, etc.) of a mesh. In many applications of scientific computing, however, the quality of a mesh is a key factor that significantly affects the numerical result of finite or boundary element analysis. One of the most common criteria for mesh quality is the uniformity of angles, although this may not be the best in some cases where anisotropic meshes are desired [1]. For its popularity, however, we shall adopt the angle-based criterion in the present work. In many real applications, the input meshes often have low quality, containing angles close or even equal to 0° or 180°. The main interest and contribution of the present work is to improve the quality of triangular surface meshes. Additionally our method will be extended to be able to remove noise and preserve sharp features on surface meshes. For simplicity, we refer to both mesh quality improvement and mesh denoising as mesh smoothing unless otherwise specified.

Mesh denoising has a long history in computer graphics and the related methods include three main categories: (1) geometric flows [2–6], (2) spectral analysis [7,8], and (3) optimization methods [9,10]. Due to its simplicity and low computational cost, Laplacian smoothing has established itself as one of the most common methods among all the geometric flow-based methods. In this method, every node is updated towards the barycenter of the neighborhood of the node. However, volume shrinkage often occurs during this process. The shrinkage problem may be tackled by methods utilizing spectral analysis of the mesh signal, which is the main idea of the second category. Optimization-based methods guarantee the smoothness of the mesh by minimizing different types of energy functions. But the iterative process searching for optimal solutions can be time-consuming.

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A variety of techniques on mesh quality improvement have been developed [11,12]. Some of the existing techniques include: (1) inserting/deleting vertices [13], (2) swapping edges/faces [14], (3) remeshing [15–18], and (4) moving vertices without changing mesh topology [19–22]. Two or more of the above techniques are sometimes combined to achieve better performance. For instance, Dyer et al. [23] integrate edge flipping, remeshing and decimation into one framework for generating high-quality Delaunay meshes. In the current work, however, we shall restrict ourselves to the methods in the last category that only adjust the nodes' coordinates. Among these methods, Laplacian smoothing in its simplest form that moves a vertex to the center or barycenter of the surrounding vertices [19] is one of the fastest methods but it may fail in improving mesh quality and is often equipped with other techniques such as optimizations [24,25]. Ohtake et al. [26] presented a method of simultaneously improving and denoising a mesh based on a combination of mean curvature flow and Laplacian smoothing. Nealen et al. [27] introduced a framework for mesh improving and denoising using Laplacian-based least-squares techniques. Both methods, as shown in [28], cannot warrant mesh quality or feature-preservation. Wang et al. [28] presented a method for mesh denoising and quality improvement by local surface fitting and maximum inscribed circles but it was heuristic and lacked mathematical foundations.

Among all the repositioning-based methods for mesh quality improvement, the optimal Delaunay triangulation (ODT) [29,1,30] has been proved to be effective on 2D triangular meshes. However, the extension from 2D meshes to 3D surface meshes is non-trivial in both mathematical analysis and algorithm design. For 3D surface meshes we need to consider not only angle quality but also mesh noise that causes bumpiness on surfaces, which was not taken into account in the original ODT method or its variants in tetrahedral mesh smoothing [31,32]. In addition, sharp surface features must be well preserved during the processes of mesh denoising and quality improvement. There have been extensive studies on feature-preserving surface mesh processing [33–38]. However, most of the previous work was focused on the mesh denoising problem but only a few dealt with both mesh denoising and quality improvement with feature preservation [28].

The main goal of the present paper is to generalize the 2D ODT idea to 2-manifold surface meshes by formulating the mesh quality improvement as an optimization problem that minimizes the interpolation error between a parabolic function and its piecewise linear interpolation at each vertex of the surface mesh. Unfortunately there is no analytical solution to this optimization problem. To solve the minimization problem faster, we consider a suboptimal problem by simplifying the objective function into a quadratic formula such that an analytical solution can be derived. The proposed suboptimal Delaunay triangulation (or S-ODT) is then extended to include two other capabilities: removing mesh noise as well as preserving sharp features on the original meshes. These two goals are achieved by using two standard techniques: curve/surface fitting [39] and local structure tensors [33].

The remainder of this paper is organized as follows. In Section 2, we extend the original ODT method [29,1] to improve the angle quality of a surface mesh. Several variants of the new algorithm are also introduced to warrant additional desirable properties such as noise removal and feature preservation. Numerous mesh examples are included and comparisons are given in Section 3 to demonstrate the performance of the proposed algorithms, followed by our conclusions in Section 4. Some mathematical details of the algorithms are provided in the Appendices.

2. Method

Like many other mesh smoothing approaches, our method is iterative and vertex-based, meaning that all mesh vertices are repositioned in each iteration and the process is repeated until the mesh quality meets some predefined criteria or a maximum number of iterations is reached. In this section we shall describe three algorithms with the basic one addressing the mesh quality improvement using the proposed sub-optimization formulation and two extended algorithms dealing additionally with the issues of feature preservation and noise removal. For completeness, we shall begin with a brief introduction to Delaunay triangulation and the original ODT method [29]. More details on ODT-based 2D/3D and local/global mesh smoothing algorithms can be found in [1,30].

2.1. Brief introduction to ODT

In computational geometry, Delaunay triangulation (DT) is a well known scheme to triangulate a finite set of fixed points P, satisfying the so-called *empty sphere condition*. That is, no point in P can be inside the circumsphere of any simplex (e.g., triangle) in $DT(P)$. Consider, for example, the four points $p0$, $p1$, $p2$ and $p3$ in Fig. 1a and b. There are obviously two ways to triangulate this point set, but only the one in Fig. 1b is a Delaunay triangulation that produces a larger minimum angle than that in Fig. 1a and thus is preferable according to the angle-based criterion. Fig. 1a and b also tells us another interpretation of Delaunay triangulation. If we lift the point set onto a parabolic function $\|\mathbf{x}\|^2$, any triangulation on the lifting points $q0$,$q1$,$q2$ and $q3$ will result in a unique piecewise linear interpolation of the parabolic function. The one that minimizes the interpolation error can be projected back to the original point set and makes the Delaunay triangulation. From this example, we can see that Delaunay triangulation of a fixed point set is equivalent to minimizing the following interpolation error, which can be achieved by swapping edges:

$$Q(DT, \|\mathbf{x}\|^2, q) = \min_{\mathcal{T}\in\mathcal{T}_p} Q(\mathcal{T}, \|\mathbf{x}\|^2, q), \quad \forall 1 \leqslant q \leqslant \infty, \tag{1}$$

where $Q(\mathcal{T}, \|\mathbf{x}\|^2, q)$ is the L^q distance between the parabolic function $\|\mathbf{x}\|^2$ and its piecewise linear interpolation $\|\mathbf{x}\|_I^2$ based on a particular triangulation $\mathcal{T}$ of a fixed point set P. $\mathcal{T}_p$ is the set of all possible triangulations of P.

Although Delaunay triangulation is optimal for a fixed set of points, it does not necessarily produce a high quality mesh if the given points are not nicely distributed. In