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Quality mesh smoothing via local surface fitting and optimum projection ☆

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ABSTRACT

The smoothness and angle quality of a surface mesh are two important indicators of the "goodness" of the mesh for downstream applications such as visualization and numerical simulation. We present in this paper a novel surface mesh processing method not only to reduce mesh noise but to improve angle quality as well. Our approach is based on the local surface fitting around each vertex using the least-square minimization technique. The new position of the vertex is obtained by finding the maximum inscribed circle (MIC) of the surrounding polygon and projecting the circle's center onto the analytically fitted surface. The procedure above repeats until the maximal vertex displacement is less than a pre-defined threshold. The mesh smoothness is improved by a combined idea of surface fitting and projection, while the angle quality is achieved by utilizing the MIC-based projection scheme. Results on a variety of geometric mesh models have demonstrated the effectiveness of our method.

1. Introduction

Surface meshes are frequently used for two primary purposes: 3-D visualization and numerical simulation. Accordingly, the "goodness" of a surface mesh is typically measured by its smoothness and angle quality. Good smoothness implies less reconstruction noise or artifact on a surface model while high-quality angles means that no angle should be close to 0° or 180°. The smoothness property is critical in 3-D visualization – noisy surface meshes often result in inaccurate interpretation of the models. On the other hand, the angle quality makes a significant impact on the approximation accuracy of numerical solutions. Ideally, meshes with uniform angles would be most desirable in simulation.

In pratice, however, many meshes fail to satisfy one or two of these requirements. For instance, triangular surface meshes generated by iso-surfacing techniques such as the Marching Cube method [29] possess good smoothness but often contain very sharp angles (see Fig. 1a), which make the meshes inappropriate for use in numerical simulation. On the other hand, quadrilateral surface meshes can be simply generated from 3-D imaging data by extracting the outward faces of segmented voxels. These meshes, if converted into triangular meshes, have very good angles (either 45° or 90°) but are so noisy that it is hard to visualize the structural details on the surface (see Fig. 1b). The goal of this paper is to present a set of efficient algorithms to process an abitrary surface mesh so that both the smoothness and angle quality will be significantly improved.

A wide variety of mesh smoothing algorithms have been proposed in recent years [2]. Most of them, however, are designed just from the graphical point of view, i.e. to improve the mesh smoothness or, in other words, to reduce mesh noise for reconstruction, rendering or visualization purposes [1]. Laplacian iterative smoothing is one of the most common and simplest techniques for mesh smoothing [17]. During each iteration, all the vertices of a mesh are adjusted to the barycenter of the neighboring region. Because of its simplicity, many variants of this method have been developed [19,7,37]. Taubin [37] proposed a mesh smoothing method by using a simple, isotropic technique to improve the smoothness of a surface

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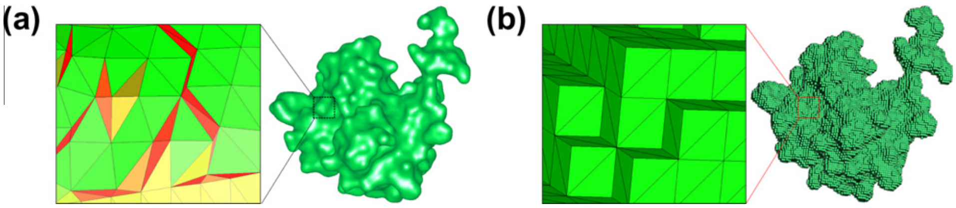

Fig. 1. (a) An example of triangular meshes generated by the marching cube method, showing very good smoothness but low angle quality. The color of a triangle is green if its minimal angle is greater than 40° and red if its minimal angle is less than 15°. Otherwise, the color is linearly interpolated from red (15°) to green (40°). (b) An example of quadrilateral meshes having uniform and regular squares but showing very noisy surfaces.

mesh. Desbrun et al. [14] extended Taubin's work to smooth irregular meshes by using geometric flows. Ohtake et al. [31] extended the Laplacian smoothing by combining geometric smoothing with parameterization regularization. Peng et al. [33] gave a denoising algorithm for geometric data represented as a semiregular mesh on the basis of adaptive Wiener filtering. Despite their high speed, these methods often yield significant volume shrinkage and undesired mesh distortion. Another popular smoothing approach is based on the energy minimization technique. Welch and Witkin [39] described an approach to designing and fairing freeform shapes represented by triangulated surfaces. Kobbelt [23,24] proposed a general algorithm to fair a triangular mesh with arbitrary topology in R^3 by estimating the curvature for the mesh model. These methods are time-consuming due to the complicated energy functions to be minimized. Recently, feature-preserving mesh smoothing methods [12,15,44,3,22,18,35,36,27] have drawn more and more attention. Bajaj and Xu [3] proposed a partial differential equation (PDE)-based anisotropic diffusion approach for processing noisy surfaces and functions defined on surfaces. Although these methods can achieve high smoothness of mesh, they rely on the formation of a "shock" term to preserve details, resulting in significant computational costs. Jones et al. [22] developed a feature-preserving smoothing algorithm by adopting local first-order predictors statistically defined on triangulated surface meshes. Fleishman et al. [18] introduced a similar method based on iterative bilateral filtering, a non-linear variation of Gaussian smoothing that weighs sample points based on their similarity to the one being processed. However, finding a set of appropriate parameters is not an easy task. Sun et al. presented fast feature-preserving mesh denoising approaches by normal-filtering and vertex-updating [36] and random walks [35]. Li et al. [27] adopted the weighted bi-quadratic Bezier surface fitting and uniform principal curvature techniques to smooth surface meshes.

Many of the methods mentioned above are devoted only to improving the smoothness of a mesh. The angle quality of a mesh is equally if not more important, especially when the numerical simulation based on the mesh is taken into consideration [45]. The quality of a mesh can be measured by either geometry-dependent [28] or solution-dependent [4] criteria. The one we use in our study is by measuring the angles of the triangulated surfaces, a strategy commonly used in the mesh generation and smoothing community [46,40]. The mesh quality may be improved by a combination of three major techniques: inserting/deleting vertices [34,13], swapping edges/faces [8,41], and moving the vertices without changing the mesh topology [17,38]. The last one, also known as mesh smoothing, is the strategy we will explore in the present paper. Different criteria can be utilized to move a vertex, including the local angle-based methods [46] and global optimization methods [9,10,30].

To enhance the smoothness of a mesh and improve its angle quality, we propose an effective mesh smoothing approach based on quadric surface fitting and maximum inscribed circle (MIC) techniques for 3-D triangular or quadrilateral meshes. As shown in Fig. 2, the basic idea of this method is to project the center of the MIC at each vertex onto the locally fitted surface and update the current vertex with the projection point to achieve both good mesh smoothness and high angle quality. Numerous experiments on biological and engineering models have demonstrated the effectiveness of our approaches in achieving these two goals. The remainder of the present paper is organized as follows. Section 2 is focused on the details of the mesh smoothing algorithm. The mesh quality improvement is described in Section 3. We present some

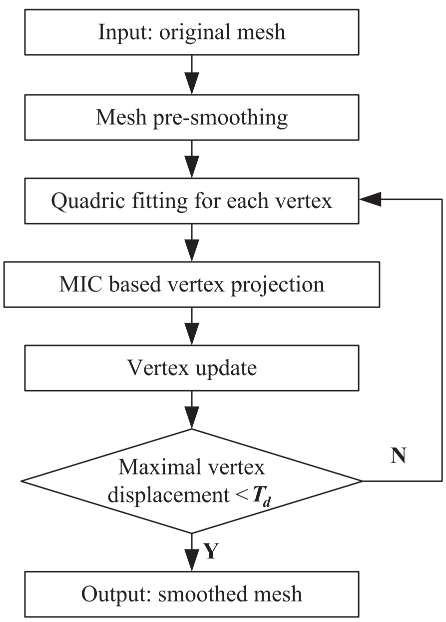

Fig. 2. The flowchart of mesh smoothing method.