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A series solution for a coaxially fed monopole in a rectangular cavity

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ABSTRACT

The boundary-value problem of radiation from a coaxially fed monopole in a rectangular cavity is considered. The image theory is invoked to transform the original problem into a simple equivalent. The boundary conditions are enforced in conjunction with the mode-matching method to obtain a set of simultaneous equations for modal coefficients. Computations are performed to check validity and numerical efficiency of the series solution.

1. Introduction

Coaxially fed monopoles and antennas in rectangular cavities, commonly used microwave structures, have been extensively studied in [1–8]. A Green's function approach was used in [1] to study radiation from a coaxially fed monopole in a rectangular cavity. The moment method was applied to model coaxial probes used in practical waveguides and cavities [2,3]. A full wave analysis and ANSOFT HFSS simulation were respectively performed to represent the input impedance of a monopole antenna placed inside the cavity in [4,5]. The transmission line model and the finite-difference method for time-domain modeling of antennas in a cavity were used in [6,7], respectively. The modal-expansion analysis combining mode matching and point matching method was performed to analyze a monopole with a delta gap source in cavities in [8]. Although the characteristics of a coaxially fed monopole in a rectangular cavity have been studied with various techniques, it is of interest to obtain a simple analytic solution without recourse to intensive numerical computation. The purpose of the present paper is to derive such a simple series solution based on the image method, the Fourier transform, and the mode-matching method [9,10]. In addition, Poisson's summation formula and residue calculus are used to represent the solution in fast convergent series that is efficient for computation. A brief theoretical summary and computation results are given next. Notations used in this paper closely follow those in [9–11].

2. Field representations

Fig. 1 shows a coaxially fed monopole in a rectangular cavity, where a monopole is located along the z-direction. The overall geometry is symmetrical with respect to the $x=0$ plane. Assume that an incident TEM (transverse electromagnetic) mode impinges on a monopole whose outer radius of a coaxial line is much smaller than the wavelength. The time convention $e^{-i\omega t}$ is suppressed throughout the analysis. We replace the original problem of Fig. 1 with an equivalent problem of Fig. 2 using the image theorem. In the equivalent problem of Fig. 2, an infinite number of coaxially fed monopoles are introduced in a parallel-plate waveguide. Note that the parallel-plate waveguide consists of two infinite conducting planes placed at $z=0$ and h_2. Herein, the $(j, u)=(0, 0)$ coaxial line is a real source, while the others ($j, u=-\infty, \ldots, -1, 0, 1, \ldots, \infty$, $j=u \neq 0$) are due to image source contributions. In order to match the boundary conditions at $x=\pm s/2$ and $y=-g$, $d-g$ in Fig. 1, we represent the magnetic field of the TEM wave incident on the (j, u) monopole of Fig. 2 as

$$H^{i_{ju}}_{\phi_{ju}}(\rho_{ju}, z) = (-1)^{j+u} H^{i_{00}}_{\phi_{ju}}(\rho_{ju}, z) \tag{1}$$

where (ρ_{ju}, ϕ_{ju}, z) is cylindrical coordinates centered at (j, u) monopole and $H^{i_{00}}_{\phi_{00}}(\rho_{00}, z)$ is the magnetic field of a incident TEM mode at (0,0) monopole of Fig. 2. In the following analysis, our formulation will utilize the mode-matching method that was developed in [9]. It is convenient to divide the region into I_{ju} ($\rho_{ju} < a_1, 0 < z < h_1$), II, and III_{ju} ($a_1 < \rho_{ju} < a_2, 0 < z < \infty$), where region II is the intersection of regions II_{ju} ($\rho_{ju} > a_2, 0 < z < h_2$) of all monopoles and j, $u=-\infty, \ldots, -1, 0, 1, \ldots, \infty$. In region III_{ju}, the total field

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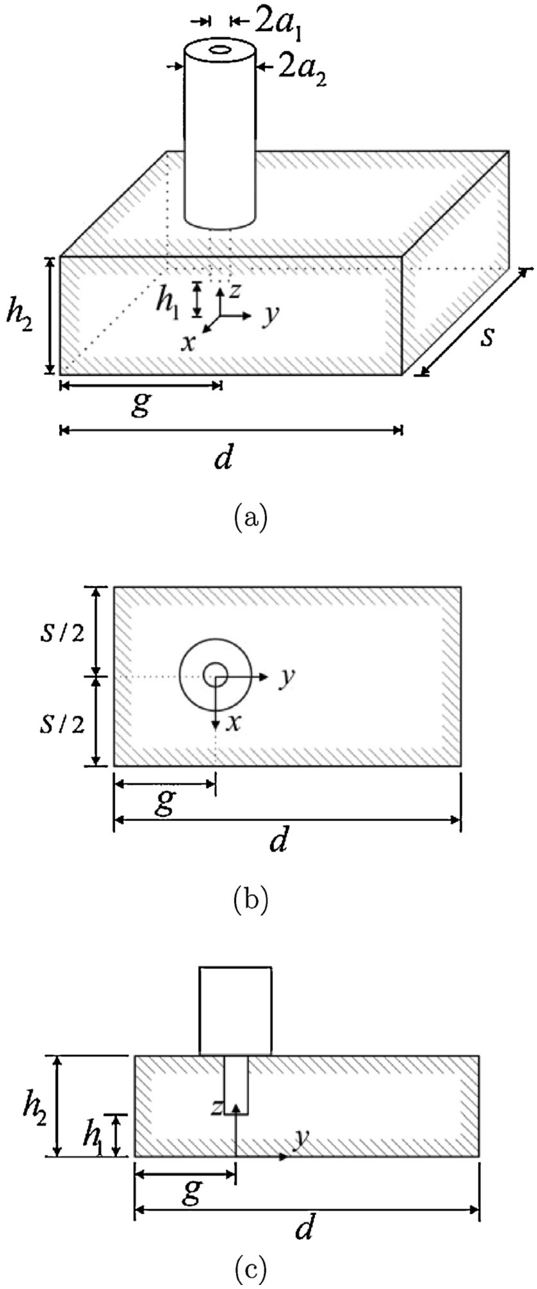

Fig. 1. Coaxially fed monopole in a rectangular cavity (a) three-dimensional view, (b) top view and (c) side view.

comprises the incident, reflected, and scattered components. They are

$$H_{\phi_{ju}}^{i_{ju}}(\rho_{ju},z) = -(-1)^{j+u}\frac{e^{-ikz}}{\eta\rho_{ju}}Ne^{i\phi} \tag{2}$$

$$H_{\phi_{ju}}^{r_{ju}}(\rho_{ju},z) = -(-1)^{j+u}\frac{e^{ikz}}{\eta\rho_{ju}}Ne^{i\phi} \tag{3}$$

$$E_z^{III_{ju}}(\rho_{ju},z) = \frac{2}{i\omega\epsilon\pi}\int_0^\infty (-1)^{j+u}R(\kappa\rho_{ju})\cos(\zeta z)d\zeta \tag{4}$$

where $E_z^{III_{ju}}(\rho_{ju},z)$ represents the z-component of the electric field in region III_{ju}. Here, N and ϕ are constants, k $(=\omega\sqrt{\mu\epsilon})$ is the wave number, $\kappa=\sqrt{k^2-\zeta^2}$, and η $(=\sqrt{\mu/\epsilon})$ is the intrinsic impedance. Note that $R(\kappa\rho_{ju})=J_0(\kappa\rho_{ju})\widetilde{E}^+(\zeta)-N_0(\kappa\rho_{ju})\widetilde{E}^-(\zeta)$, where $\widetilde{E}^+(\zeta)$ and $\widetilde{E}^-(\zeta)$ are the unknown coefficients, and $J_0(\cdot)$ and $N_0(\cdot)$ are the Bessel functions of the first and second kinds, respectively. In regions I_{ju} ($\rho_{ju}<a_1$, $0<z<h_1$) and II_{ju} ($\rho_{ju}>a_2$, $0<z<h_2$), we represent the scattered electric fields in terms of the unknown discrete modal coefficients p_m and q_m. They are

$$E_z^{I_{ju}}(\rho_{ju},z)=\frac{i}{\omega\epsilon}\sum_{m=0}^{\infty}(-1)^{j+u}p_m\xi_{1m}\cos(h_{1m}z)J_0(\xi_{1m}\rho_{ju}) \tag{5}$$

$$E_z^{II_{ju}}(\rho_{ju},z)=\frac{i}{\omega\epsilon}\sum_{m=0}^{\infty}(-1)^{j+u}q_m\xi_{2m}\cos(h_{2m}z)H_0^{(1)}(\xi_{2m}\rho_{ju}) \tag{6}$$

where $h_{pm}=m\pi/h_p$, $\xi_{pm}=\sqrt{k^2-h_{pm}^2}$, and $H_0^{(1)}(\cdot)$ is the zeroth-order Hankel function of the first kind. In view of superposition principle, we represent the total electric field in region II (intersection of regions II_{ju} of all monopoles) of Fig. 2 approximately as

$$E_z^{II}=\sum_{u=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}E_z^{II_{ju}}(\rho_{ju},z) \tag{7}$$

3. Boundary conditions

Our aim is to constitute a set of simultaneous equations for the unknown discrete modal coefficients p_m and q_m. The boundary conditions need to be enforced to determine p_m and q_m. Thanks to the symmetry, we only need to apply the boundary condition at a real monopole ($\rho_{00}=a_1, a_2$). Applying the Fourier cosine transform $[\int_0^\infty(\cdot)\cos(\zeta' z)dz]$ to the tangential electric field (E_z) continuity at $\rho_{00}=a_1$, we get

$$R(\kappa a_1)=-\sum_{m=0}^{\infty}p_m\xi_{1m}J_0(\xi_{1m}a_1)F_m^1(\zeta) \tag{8}$$

where

$$F_m^p(\zeta)=\frac{(-1)^m\zeta\sin(\zeta h_p)}{\zeta^2-h_{pm}^2} \tag{9}$$

Similarly, applying the Fourier cosine transform to the tangential electric field (E_z) continuity at $\rho_{00}=a_2$, we get

$$R(\kappa a_2)=-\sum_{m=0}^{\infty}q_mS_{0m}\xi_{2m}F_m^2(\zeta) \tag{10}$$

where

$$\begin{aligned}S_{pn} &= H_p^{(1)}(\xi_{2n}a_2)+J_p(\xi_{2n}a_2)\sum_{\substack{u=-\infty\\u\neq 0}}^{\infty}(-1)^uH_0^{(1)}(\xi_{2n}|u|s)\\ &\quad +J_p(\xi_{2n}a_2)\sum_{u=-\infty}^{\infty}\sum_{\substack{j=-\infty\\j\neq 0}}^{\infty}(-1)^{j+u}H_0^{(1)}(\xi_{2n}d_{ju})\end{aligned} \tag{11}$$

Here d_{ju} is the distance between the $(0,0)$ and (j,u) monopoles of Fig. 2, and $H_p^{(1)}(\cdot)$ and $J_p(\cdot)$ are the Hankel function of the first kind and the Bessel function of the first kind, respectively. Graf's addition theorem [12] was used. Note that S_{pn} represents the contribution of each coaxial line to the (0, 0) coaxial line. The fast convergent series form of S_{pn} is summarized in Appendix A.

We next utilize the tangential magnetic field (H_ϕ) continuity and sinusoidal orthogonality to obtain a set of simultaneous equation