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Performance evaluation of a new variable tap-length learning algorithm for automatic structure adaptation in linear adaptive filters

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ABSTRACT

In this work, a variable-tap length, variable step normalized least mean square algorithm with variable error spacing is proposed. The algorithm finds the optimized tap-length that best balances the complexity and steady state performance in linear adaptive filters. The design provides a systematic procedure with mathematical analysis to select the variable key parameters that affect the structure adaptation. The proposed structure adaptation algorithm maintains a trade-off between the mean square error and convergence speed. A sliding window weight update method is presented along with the tap-length learning algorithm to reduce the structural as well as computational complexity. Guidelines for parameter selection to formulate the optimum tap-length in correspondence with the designed algorithm are shown and assumptions are specified. The proposed algorithm has performed better than the existing fractional tap-length learning methods for both low and high noise conditions. This is achieved because of the unique method adopted in this paper to set dynamic system independent parameters instead of predefined fixed settings.

1. Introduction

Both the tap-length and step-size play an important part in balancing the overall performance of the tapped delay line structure of the adaptive filter. The structural design complexity in adaptive filter largely depends on its tap-length while the convergence speed depends on the proper selection of error spacing and of step-size parameters [1–4]. The steady state performance of least mean square (LMS) in the context of mean square error (MSE), convergence, computational complexity is governed by the weight updating step size as well as the structural design [5]. The tap-length is fixed at a compromise value during system identification analysis which may result in an inefficient design [6,7].

In many applications, the tap-length that best adjust the equalizer performance varies with time to combat the time varying multipath channel effect [8]. The first variable tap-length algorithm [2] justified that filters with fewer taps has faster convergence speed than the larger ones and adjustment of the tap-length can improve the convergence of the LMS [3,9]. The current MSE is compared to the previously estimated minimum mean square error (MMSE) to find the cost function for variable tap-length selection. The research presented in [8] is based on segmented filtering approach which suffers from the drawback of careful selection of the segment order and use of absolute error instead of MSE. Based on gradient descent approach another algorithm is proposed [10] where tap-length is adjusted dynamically along the negative direction of the squared estimated error signal. It was proved to be more flexible than the segmented approach but created the issues of wandering. The fractional tap-length LMS (FT-LMS) [1,11] design established a well defined approach for tap-length adaptation but random choice of parameters like error spacing, leakage factor, adaptation step size, weight update step-size limits its applicability. Here setting different thresholds for cost function results in different performance analysis. Again the error spacing parameter can be varied with tap-length variation to achieve better convergence behaviour and small steady state error [12]. An improved FT-LMS algorithm is proposed and steady state performance analysis is done in [13]. This provides a guideline for parameter choice but fails to address all the issues with FT-LMS. In [4,14] a variable tap-length variable step LMS (VT-VSLMS) algorithm has been proposed which performs better than FT-LMS algorithm but it suffers from random selection of a leaky factor which controls the structure adaptation without any clear mathematical analysis. A variable leakage factor FT-LMS algorithm [15] proposed the use of a variable leakage parameter but employed simple LMS algorithm, a fixed error width parameter and random choice of fixed adaptation step-size which may affect the convergence speed and MSE performance. In many

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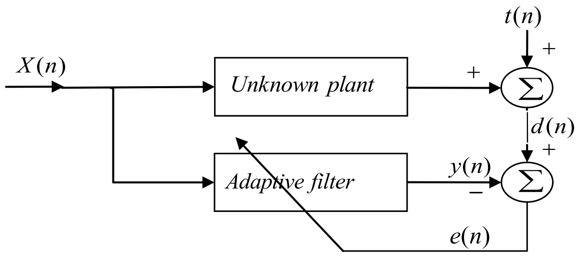

Fig. 1. Adaptive filter modelling module.

practical applications, the peak numerical complexity of the adaptive algorithms is considered to be more important that the average complexity.

In this paper an improved algorithm is presented for optimum tap-length selection in a time varying environment. The algorithm analysis is made to find the variable parameters that best adjust the steady state performance as well as the convergence speed in a time varying environment under various noise conditions. Importance is given to fix the parameters that affect the structure adaptation as per the variation in the length of adaptive filter. In existing algorithms, the parameters are of fixed value and system dependent. They work only for low noise conditions and the pseudo fractional system distance was not studied for computational complexity issue. The weight update equation was either LMS or any modified version of it like normalized LMS (NLMS), variable step LMS (VSNLMS). In proposed algorithm a new sliding window weight update is applied to reduce the computational along with the structural complexity. All key parameters like weight, input signal, error signal, error width and desired signal are made dependent on tap-length variation which results in an improved overall performance both in convergence and steady state error.

2. Variable tap-length learning for linear adaptive filters

In Fig. 1 a simple adaptive system identification model is shown. An unknown plant has to be identified by an adaptive filter employing a dynamic tap-length selection algorithm. Again selecting the tap-length for any system identification frame work is not a trivial task. The selection depends on nature of the system to be identified, memory requirement, desired performance, computational complexity, noise level and parameter variation, etc. In most filter designs for different unknown plants the tap-length is unfortunately fixed at some compromise value creating the problem of too short and too long filters which results in under-modelling and slow convergence respectively. So length of the adaptive filter must vary and approach to optimal value in the time varying scenarios to best balance the system identification performance. Practically in an identification framework the length of unknown plant might be very large or small. This results in issues like adaptation noise due to mismatch of extra taps and system under-modelling because of unavailability of taps [7]. The requirement is to construct a suitable dynamic tap-length selection algorithm not only to minimize the structural and computational complexity of the filter design but also to improve the overall system identification performance.

In [1] the optimum order is defined as L_{opt} that satisfies,

$$Q_{L-1}(\infty) - Q_L(\infty) \le \delta \quad \text{for all} \quad L \ge L_{\text{opt}} \tag{1}$$

where $Q_{L-1}(\infty) - Q_L(\infty)$ is the difference between the converged MSE when the tap-length is increased from $L-1$ to L and δ is a very small positive number which is set pertaining to the system requirement. The cost function with respect to tap-length L was defined as,

$$\min\{L | Q_{L-1} - Q_L \le \delta\} \tag{2}$$

Considering the error spacing parameter $\varDelta$ the modified cost function for searching the optimum tap-length can be defined as [4]

$$\min\{L | Q_{L-\varDelta} - Q_L \le \delta\} \tag{3}$$

Neglecting the adaptation noise $Q_{L-\varDelta}$ and Q_L corresponds to the MMSE for $L-\varDelta$ and L and taps respectively and [4,14]

$$Q_{L-\varDelta} - Q_L \ge Q_{L-1} - Q_L \tag{4}$$

The equality holds well when $Q_{L-1} = Q_L$ [1,4]. In the fractional tap-length estimation the error width $\varDelta$ plays an important role. It provides the trade-off between the convergence rate and steady state bias [12]. The variable time error width $\varDelta(n)$ concept was introduced in [12] and modified in [4]. It is obtained as

$$\varDelta(n) = \max(\varDelta_{\min}, \varDelta_{\max}\hat{S}) \tag{5}$$

where $\hat{S}$ is smoothing factor whose value changes according to its maximum and minimum limit [4,12,14]. The weight is updated in FT-LMS and VT-VLMS as per simple NLMS or VSNLMS algorithm,

$$W_{L(n)}(n+1) = W_{L(n)}(n) + \mu(n) X_{L(n)}(n) e(n) \tag{6}$$

where $W_{L(n)}(n), X_{L(n)}(n)$, are the weight and input vector respectively pertaining to time varying tap-length $L(n)$, $e(n)$ is the error signal and $\mu(n)$ is the time varying step size. The pseudo fractional tap-length that automatically does the structure adaptation in a dynamic time varying situation was first obtained efficiently by following the adaptation proposed [1,15]

$$L_{nf}(n+1) = [L_{nf}(n) - \Psi] + [Q_L - Q_{L-\varDelta}]\bar{\Psi} \tag{7}$$

Finally the tap-length $L(n+1)$ in the adaptation of filter weights for next iteration is formulated as follows [1],

$$\begin{aligned} L(n+1) &= \left\langle L_{nf}(n) \right\rangle \quad \text{if} \left| L(n) - L_{nf}(n) \right| > \frac{\Psi}{\bar{\Psi}} \\ &= L(n) \quad \text{otherwise} \end{aligned} \tag{8}$$

$L_{nf}(n)$ is the tap-length which can take fractional values. $Q_L - Q_{L-\varDelta}$ is the MSE difference with $\varDelta$ error spacing [1]. The $\langle\rangle$ is used to round of the actual fractional order of the adaptive filter $L_{nf}(n)$ to the nearest integer value. In (7) the factor Ψ is the leakage factor which prevents the order to be increased to an unexpectedly large value and $\bar{\Psi}$ is the step size for tap-length adaptation. The fractional order estimation algorithms converge in mean. The proposed algorithm is discussed in context with all these algorithms while resulting in a more efficient way to find the optimum tap-length for any system identification framework.

3. The algorithm analysis

3.1. VSNLMS sliding window weight update

In this paper the design problem is related with optimizing tap-length related criterion in an identification framework of adaptive filter. This research is based on a low complexity design of adaptive filter. It aims to minimize the structural complexity by optimizing the tap-length, hence follows an advanced variant of LMS algorithm for weight update instead of recursive least square algorithm which presents excellent system identification performance, but at the same time brings heavy computational complexity ($O(N^2)$). Another reason for employing LMS type algorithm is that all existing structure adaptation algorithms are based on LMS and its modern variants. In this work a variable-tap length, variable step normalized least mean square algorithm with variable error spacing (VT-VSNLMS$_{\text{VE}}$) is described within a system identification model in which both the optimum tap-length $L_{\text{opt}}(n)$ and coefficient of the unknown system pertaining to that $W_{L_{\text{opt}}}(n)$ are to be