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Analysis of Fraction Skill Score properties for a displaced rainy grid point in a rectangular domain

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ABSTRACT

The Fraction Skill Score (FSS) is a recently developed and popular metric used for precipitation verification. A compact analytical expression for FSS is derived for a case with a single displaced rainy grid point in a rectangular domain. The existence of an analytical solution is used to determine some properties of FSS, which might also be applicable in other cases since the rain areas of any shape will asymptote towards this solution if the displacement is sufficiently large. The use of the simple square shape of the neighborhood causes the FSS value to be dependent on the direction of the displacements (not only on the displacement size). The effect is limited in scope but can increase or decrease the FSS value by 0.1. Moving a nearby border closer to the rainy points can either increase or decrease the FSS value, depending on the location of the border. The FSS value near a border can be at most 33% larger than the FSS value in the infinite domain, assuming the same neighborhood size and displacement. The effect of the nearby corner is similar to the effect of the nearby border but is stronger. The useful forecast criteria (FSS_{useful}) is defined as a value of FSS for a precipitation feature with a displacement half the neighborhood size. FSS_{useful} for a displaced rainy grid point depends on the orientation of the displacement being the largest for displacements that are parallel to the borders and the smallest for a diagonal displacement for which the value can be as low as 0.42. An analysis of a real dataset was also performed, which showed that the border effect is usually small, but in some cases the effect becomes large (an increase of FSS value up to 70% was identified). The likelihood of a strong border effect in real datasets increases significantly if the neighborhood size at FSS = 0.5 is comparable or larger than the domain size.

1. Introduction

The Fraction Skill Score (FSS) (Roberts and Lean, 2008; Roberts, 2008) is a recently developed and popular metric used for precipitation verification (Ebert et al., 2013). Many studies use the FSS for verification of precipitation (e.g. Wang et al., 2013a, 2013b; Romine et al., 2013; Baldauf et al., 2011; Sobash et al., 2011; Rennie et al., 2011). The method is relatively simple to understand and easy to implement while simultaneously exhibiting some very useful properties. One of its most valuable properties is the ability of the FSS to determine the spatial scale at which the forecast can be deemed useful (Ebert et al., 2013; Rossa et al., 2008).

Although many studies have used FSS for precipitation verification, there are very few studies focusing on analysis of basic FSS properties. These include the original paper by Roberts and Lean (2008) that introduced the FSS and some of its basic properties along with the useful forecast criteria; Roberts (2008) who introduced the use of frequency thresholds to remove the bias and linked the useful forecast scale to the displacement in an idealized experiment; and Mittermaier and Roberts (2010) that showed that the FSS can provide a truthful assessment of displacement errors for some geometric cases.

In fact, the properties of the FSS are not yet fully understood, especially regarding the effects of differently shaped precipitation areas and the positions of these areas in relation to the domain borders. This study is continuing the research by Skok (in press), in which an analysis of FSS properties using an idealized case with a displaced rain band was performed. The displaced rain band case was first analyzed numerically in the original FSS paper (Roberts and Lean, 2008) and later in Duc et al. (2013). In contrast, this study focuses on FSS properties of a displaced rainy grid point in a rectangular domain. A complete analytical solution is presented, valid for all possible rectangle shapes of the domain and all possible positions of the rainy point and its displacement. The existence of an analytical solution will be used to determine some properties of the FSS that are also applicable in other cases. It is reasonable to assume that the FSS value of displaced rain area of any shape will asymptote towards the rainy grid point solution if the displacement is sufficiently large (displacement much larger than the rain area size). In the rain

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band analysis of Skok (in press), the solution was limited to rain bands oriented parallel to one of the borders and displacements that were perpendicular to rain band orientation (otherwise, obtaining an appropriately compact analytical solution would not be possible). The solution for a displaced rainy grid point is more general, thus allowing for arbitrary displacements. This enables the study of influence of the direction of the displacement on the FSS value in addition to the influence of the size of displacement. Moreover, the influence of the nearby corners can be analyzed.

In the second section, the analytical expression for FSS in an infinite domain is derived, while the solution for a rectangular domain is presented in the third section. The fourth section studies the effect of the square shape of the neighborhood, while in the fifth section the displacement parallel to the borders is analyzed. The sixth section studies the influence of one nearby border while the influence of a nearby corner is studied in the seventh section. The eighth section analyzes the criteria for a useful forecast, and the ninth section analyzes real datasets.

2. Solution for an infinite domain

An assumption is made that a single rainy grid point was observed in an otherwise empty infinite domain using a regular rectangular grid (Fig. 1a). The single rainy grid point is also present in the forecast but is displaced by v grid points in the vertical direction and h grid points in the horizontal direction (if $v = 0$ and $h = 0$ there is no displacement). Since the assumption is made that the domain is infinitely large, any effect the domain borders might have on the FSS value is eliminated. A square neighborhood of size $n \times n$ grid points is used (n is an odd number). The vertical direction is indicated by index j and the horizontal by index i.

The first step is to define the expression for fractions. The fractions are portions of the neighborhood covered by precipitation. They can only have two possible values since the rainy grid point can either be inside the neighborhood (fraction value $1/n^2$) or outside the neighborhood (fraction value 0). As a result, the domain can be separated into four regions (Fig. 1b): the inner region (denoted with number 2 in Fig. 1b) where observation and forecast fractions are both nonzero, the two outer regions where one fraction is nonzero while the other one is zero (denoted with numbers 1 and 3),and an external region where both are zero (denoted with number 0). Inside each of these regions, the observation and forecast fractions are constant.

According to the definition in Roberts and Lean (2008), the FSS can be calculated as

$$\mathrm{FSS} = 1 - \frac{\frac{1}{N}\sum_i\sum_j[O(i,j)-M(i,j)]^2}{\frac{1}{N}\sum_i\sum_j O(i,j)^2 + \frac{1}{N}\sum_i\sum_j M(i,j)^2}$$

where N is the total number of grid points in the domain, and the double sum represents the sum over all grid points in the domain. Using some simple mathematical operations, the FSS can be rewritten in a more compact form as

$$\mathrm{FSS} = \frac{2\sum_i\sum_j O(i,j)\cdot M(i,j)}{\sum_i\sum_j O(i,j)^2 + \sum_i\sum_j M(i,j)^2} = \frac{2A}{B+C} \tag{1}$$

where $A = n^4\sum_i\sum_j O(i,j)\cdot M(i,j)$, $B = n^4\sum_i\sum_j O(i,j)^2$ and $C = n^4\sum_i\sum_j M(i,j)^2$. Since the fraction values can only be 0 or $1/n^2$, the A, B and C are non-negative integer numbers. The value of A represents the area (in number of grid points) where both observational and forecast fractions are non-zero (region 2 in Fig. 1b). The value of B represents the area where observational fractions are non-zero (region 1 + region 2) and value of C represents the area where forecast fractions are non-zero (region 3 + region 2). The areas represented by A, B and C are all rectangular, and their values can easily be determined with the help of Fig. 1b as $A = (n - v)(n - h)$ and $B = C = n^2$. Inserting this into Eq. (1) yields.

$$\mathrm{FSS} = \frac{(n-v)(n-h)}{n^2}. \tag{2}$$

Since the value of A represents an area of a rectangular region, the width and height of this region has to be non-negative. This requires that the $n - v \geq 0$ and $n - h \geq 0$ conditions be satisfied in order for Eq. (2) to be valid. Therefore, the complete solution can be written as:

$$\mathrm{FSS} = \begin{cases} \frac{(n-v)(n-h)}{n^2};\ n \geq v \text{ and } n \geq h \\ 0\ ;\text{otherwise} \end{cases}. \tag{3}$$

Analysis of Eq. (3) shows that increasing v or h can only decrease the FSS value while increasing n can only increase the FSS value. This can be

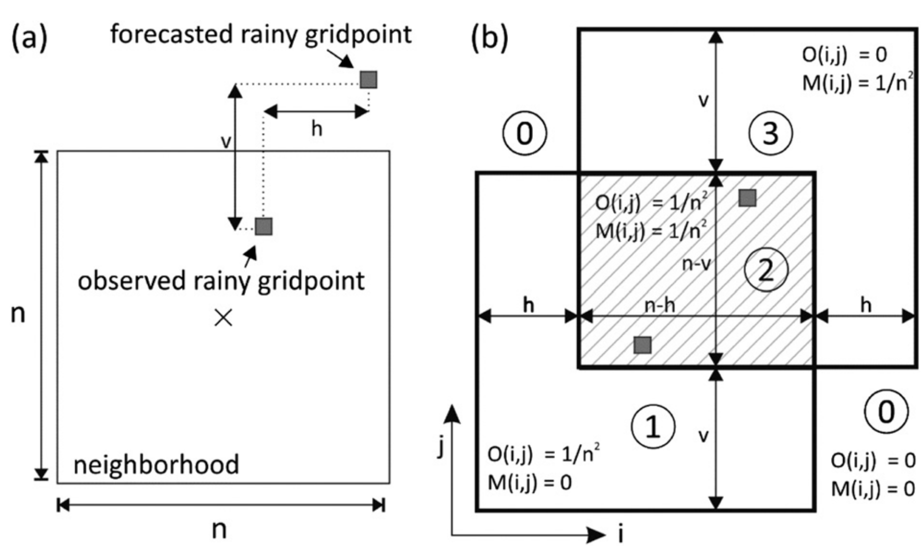

Fig. 1. a) A displaced rainy grid point and square neighborhood, b) separation of the domain into four regions (marked with numbers in circles: 0 — external region, 1 — left outer region, 2 — inner region (area marked with hatched gray diagonal lines), 3 — right outer region) and the corresponding values of observation/forecast fractions.