Int. J. Electron. Commun. (AEÜ) 69 (2015) 408–417

Contents lists available at ScienceDirect

International Journal of Electronics and Communications (AEÜ)

journal homepage: www.elsevier.com/locate/aeue

Linear programming applied to blind signal equalization

Marcelo A.C. Fernandes *

Department of Computer Engineering and Automation, Center of Technology, Federal University of Rio Grande do Norte – UFRN, Natal, Brazil

ARTICLE INFO

Article history:
Received 29 April 2014
Accepted 20 October 2014

Keywords:
Blind equalization
Linear programming
Convex function

ABSTRACT

The aim of this paper is to present a proposal for convex optimization, based on linear programming, for blind equalizers applied to digital communication systems. Different to solutions that have been presented in the literature using linear programming, the proposed model takes into consideration channels with intersymbol interference and Gaussian additive noise. The work also provides a comparative evaluation of the performance of interior-point, active-set, and simplex methods applied to the optimization process involving the blind equalizer. The results of simulations for different digital communication systems are presented using bit error rate performance curves.

1. Introduction

The signals in wireless communication systems can be corrupted by a variety of factors, notably thermal noise and multi-path phenomena that can cause frequency-selective fading. Thermal noise, which is modeled by random variables with a given probability distribution, can be efficiently minimized using channel encoders that use redundancy symbols for reconstruction of the transmitted signal. However, it is important to note that multi-path phenomena, caused by diverse reflections of the signal during transmission, are not effectively treated by channel encoders [1,2].

Multi-path phenomena are the main agents responsible for the appearance of intersymbol interference (ISI), characterized by the superposition of symbols from the same source in the domain of time. In order to minimize the effects of ISI, a variety of devices can be used in the reception process, amongst which are linear equalizers. These equalizers are filters able to compensate the non-ideal channel response, in order to recover the transmitted signal. ISI is highly dynamic and changes according to the environment, so it is necessary to employ efficient adaptive algorithms in the equalizers. In turn, the adaptive algorithms adjust the coefficients of the equalizers (the parameters to be optimized) in order to attenuate the ISI [1–3].

Amongst the various classes of adaptive equalizers are the blind equalizers [4–6], which do not require the transmission of a reference signal to the receiver in order to calculate the coefficients. Blind equalizers minimize the problem of ISI using observation of the output signal of the communication channel, together with certain a priori information concerning the statistical properties of the transmitted signal. The fact that no reference signal is used increases the data transmission capacity in systems employing blind equalization. A variety of techniques can be used for the adaptation (or optimization) of the coefficients of the blind equalizer [4–7], and one of the most widely used is the gradient descent optimization method known as the constant modulus algorithm (CMA) [5,7]. Nonetheless, the optimization function of the CMA does not provide full convergence, and presents undesirable local minima that can lead to inefficient reduction of ISI [5,7].

In order to improve the performance of blind equalizers when faced with problems of local minima, the work presented in [8] proposed a globally convergent blind equalization technique based on a convex cost function. Following the guidelines concerning the convex cost function given in [8], the work presented in [9] described a linear programming (LP) methodology applied to blind equalizers, as an alternative to the CMA algorithm. Other work concerning the utilization of linear programming associated with blind equalization is described in [10–12]. In the work presented in [13–17] LP is used in an equalizer associated with the channel coding process.

However, the earlier work [8–10] concerning the proposal for blind equalization based on linear programming was incomplete in several ways. These include the lack of any analysis of the performance of the technique in channels with ISI and additive white Gaussian noise (AWGN), no analysis of performance compared to the CMA algorithm, and the absence of tests and simulations using other LP optimization algorithms. Given these considerations, the present paper contributes to the refinement of the technique proposed in [9], making new analyses and proposing a new constraint function and other LP optimization algorithms to be applied in a

* Tel.: +55 8432153771.
E-mail address: mfernandes@dca.ufrn.br

http://dx.doi.org/10.1016/j.aeue.2014.10.017
1434-8411/

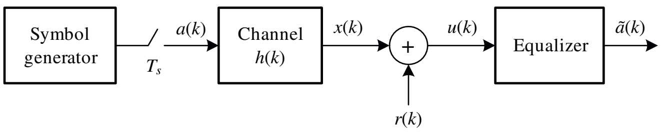

Fig. 1. Discrete baseband communication system with ISI and AWGN.

blind adaptive equalizer that is here called the BLE-LP (blind linear equalizer based on linear programming).

2. Characterization of the communication channel

Fig. 1 presents a simplified structure of a discrete baseband digital communication system with ISI (represented by $h(k)$), thermal noise (represented by $r(k)$), and a source of information, transmitting complex symbols, $a(k)$, belonging to a set, $A=\{a_0, \ldots, a_{M-1}\}$, of M possible symbols. The symbols are transmitted with a sampling period of T_s seconds (symbol interval), and each symbol is represented by words of b bits.

The complex symbols, $a(k)$, are expressed by

$$a(k) = a^I(k) + ja^Q(k), \tag{1}$$

where $a^I(k)$ and $a^Q(k)$ are the unidimensional phase components and quadrature, respectively, that constitute the bidimensional transmitted signal. The symbols $a(k)$ are transmitted by means of an impulse response channel, $h(k)$, and are subject to the action of ISI and complex AWGN, $r(k)=r^I(k)+jr^Q(k)$, as illustrated in Fig. 1. The signals $r^I(k)$ and $r^Q(k)$ are random circular variables with a Gaussian distribution, with an zero mean and variance σ_r^2 [2].

As shown in Fig. 1, the linear equalizer processes the signal, $u(k)$, resulting from the sum of the channel output and the AWGN, expressed as

$$u(k) = x(k) + r(k), \tag{2}$$

$$x(k) = \rho_0(k)a(k-\tau_0(k)) + \sum_{i=1}^{L-1}\rho_i(k)a(k-\tau_i(k)), \tag{3}$$

where L is the number of paths of the channel, ρ_i is the complex gain of the ith path, and $\tau_i(k)$ is an integer representing the delay of the ith path at instant k.

3. Blind adaptive equalization

The objective of a linear equalizer is to reduce the ISI component in the received signal, $u(k)$ [2,3]. Its structure is illustrated in Fig. 2, and the output signal can be expressed by

$$\begin{aligned}\tilde{a}(k-d_{eq}) &= \sum_{l=0}^{N-1} w_l(k)u(k-l)\\ &= \sum_{l=0}^{N-1} w_l(k)x(k-l) + \sum_{l=0}^{N-1} w_l(k)r(k-l),\end{aligned} \tag{4}$$

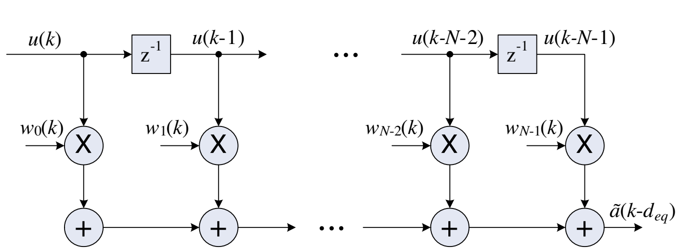

Fig. 2. Structure of a linear equalizer of length N.

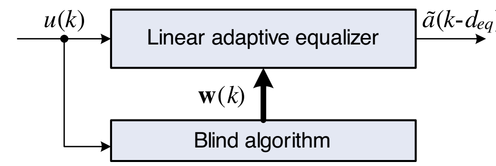

Fig. 3. Structure of a blind linear adaptive equalizer.

where w_l is the lth complex gain of the equalizer, and d_{eq} is the equalization delay.

The linear adaptive equalizer (Fig. 3) is a linear digital filter (as shown in Fig. 2), which operates in conjunction with an algorithm in order to adapt its parameters, $\mathbf{w}(k)$, according to the random variation of the impulse response of the communication channel. The adaptation proceeds by optimizing a cost function, $J(\mathbf{w}(k))$, in which

$$\begin{aligned}\mathbf{w}(k) &= \begin{bmatrix} w_0(k) \\ \vdots \\ w_{N-1}(k)\end{bmatrix} = \begin{bmatrix} w_0^I(k) \\ \vdots \\ w_{N-1}^I(k)\end{bmatrix} + j\begin{bmatrix} w_0^Q(k) \\ \vdots \\ w_{N-1}^Q(k)\end{bmatrix}\\ &= \mathbf{w}^I(k) + j\mathbf{w}^Q(k),\end{aligned} \tag{5}$$

where $\mathbf{w}^I(k)$ and $\mathbf{w}^Q(k)$ are real and imaginary parameters of the equalizer, respectively.

There are two families of adaptive equalizers, either supervised or blind, which differ in the way that the adaptation algorithm operates [3]. The blind equalization algorithms do not require a training sequence, and use statistical metrics of the transmitted signal itself in order to adjust the parameters [4–7]. One of the most well-known blind equalization algorithms is the constant modulus algorithm (CMA) [5,7].

The CMA attempts to adjust a power integer, p, of the information leaving the adaptive filter to a real positive constant, r_p. This constant is selected in order to project onto a circle all the points of the output constellation of the adaptive filter [5,7]. The cost function to be optimized, J_{CMA}, is expressed by

$$J_{CMA}(\mathbf{w}(k)) = E[e(k)^2], \tag{6}$$

where $E[\cdot]$ is the mean operator, and

$$e(k) = \tilde{a}(k)\left(\gamma - |\tilde{a}(k)|^2\right), \tag{7}$$

in which γ is the dispersion constant given by

$$\gamma = \frac{E\left\{|a_k|^4\right\}}{E\left\{|a_k|^2\right\}}, \tag{8}$$

where a_k belongs to the set A of possible modulation symbols employed. The cost function, J_{CMA}, is optimized by the gradient descent method with the classical stochastic approximation (substituting the mathematical expectation for an instantaneous estimate) [3]. Nevertheless, as shown previously [18,19,5,7], the Godard criterion, used in the CMA algorithm, possesses points of local minima that can hinder the equalization process.

4. Blind linear equalizer based on linear programming

4.1. Linear programming

There are several algorithms that can be applied to linear programming problems. These include the interior-point, active-set, and simplex methods, which have a wide range of applications.