Atmospheric Research 150 (2014) 79–96

Contents lists available at ScienceDirect

Atmospheric Research

journal homepage: www.elsevier.com/locate/atmos

Numerical simulation of the flow fields around falling ice crystals with inclined orientation and the hydrodynamic torque

Tempei Hashino [a,*,1], Mihai Chiruta [b], Dierk Polzin [a], Alexander Kubicek [a], Pao K. Wang [a]

[a] Department of Atmospheric and Oceanic Sciences, University of Wisconsin-Madison, Madison, WI, USA
[b] AerosolWorks, LLC, Madison, WI, USA

ARTICLE INFO

Article history:
Received 14 May 2014
Received in revised form 4 July 2014
Accepted 4 July 2014
Available online 19 July 2014

Keywords:
Ice crystal
Flow field
Hydrodynamic torque
Numerical solution
Navier–Stokes equation

ABSTRACT

The flow field and orientation of ice particles are fundamental information to understand cloud microphysical processes, optical phenomena, and electric-field induced orientation and to improve remote sensing of ice clouds. The purpose of this study is to investigate the flow fields and hydrodynamic torques of falling ice columns and hexagonal plates with their largest dimension inclined with respect to the airflow. The Reynolds numbers range from 2 to 70 for columns and 2 to 120 for plates. The flow fields are obtained by numerically solving the relevant Navier–Stokes equations under the assumption of air incompressibility. It was found that for the intermediate Reynolds number the streamlines around the inclined crystals exhibit less spiral rotation behind them than those around the stable posture. The vorticity magnitude was larger in the upstream side and broader in the downstream than the one without inclination. For plates, a high-pressure dome on the center of the lower basal face disappears with inclination, possibly leading to an increase of riming there. The torques acting on the crystals have a local maximum over the inclined angle and exhibit almost symmetric around 45° over the range of Reynolds numbers. The torque parameterization was performed under pressures of 300, 500, and 800 hPa as a function of Reynolds number and aspect ratio. It was found that the time scale of rotation for plates is smaller than the one for columns. Furthermore, the torque formula was applied to assess alignment of crystals along electric fields. It was found that these crystals of millimeter size require 120 kV/m for the electrical alignment, which agrees with previous studies.

1. Introduction

The flow fields and orientations of cloud and precipitation particles play an important role in cloud microphysics. The flow field around a single or multiple ice particles is inherently complicated due to the shape. One of the relevant growth processes is the collision growth of ice particles. It depends on how a small droplet or another ice crystal is moving about the target ice crystal, which is controlled mainly by the flow field. Wang and Ji (2000) investigated the riming efficiencies of small ice crystals numerically, and showed that the collision cross sections depend on the crystal shapes and droplet sizes. The flow field is also important for depositional growth through which the vapor density can be locally enhanced (e.g., Ji and Wang, 1999). This is called the ventilation effect. Another importance for investigating the flow field and orientation lies in understanding optical phenomena and application for remote sensing. Westbrook (2011) reported that the scalene columns could orient with the two prism facets laid horizontally, which may be responsible for the Parry arc. The specular reflection of hexagonal plates is often observed with lidar in ice clouds and mixed-phase clouds from the ground (e.g., Noel et al., 2002; Westbrook et al., 2010) and from space (e.g., Hu, 2007). Specular reflection and the assumption on the titling of

* Corresponding author at: Research Center for Environmental Changes, Academia Sinica, 128 Academia Road, Sec. 2, Nankang, Taipei 115, Taiwan.
E-mail address: hashino@gate.sinica.edu.tw (T. Hashino).
[1] Current affiliation: Research Center for Environmental Changes, Academia Sinica, Nankang, Taipei, Taiwan.

http://dx.doi.org/10.1016/j.atmosres.2014.07.003
0169-8095/

plates play a role in ice microphysics retrievals (Okamoto et al., 2010). Also, ice crystals are typically aligned with the electric fields in thunderstorms (e.g., Vonnegut, 1965; Hendry and McCormick, 1976; Ryzhkov and Zrnic, 2006) and the alignment has been experimentally investigated for small crystals (<30 μm) (Foster and Hallett, 2002, 2008). Such information is potentially useful for severe weather nowcasting (e.g., Krehbiel et al., 1996; Schultz et al., 2011).

Numerical methods have been a powerful tool to study the flow fields around non-spherical particles such as 2D circular cylinders (e.g., Thom, 1933; Dennis and Chang, 1969, 1970; Hamielec and Raal, 1969; Takami and Keller, 1969) and axisymmetric oblate spheroids (Rimon and Lugt, 1969; Masliyah and Epistein, 1970). In order to investigate collision efficiency with supercooled water drops, Schlamp et al. (1975) simulated the flow fields around falling ice columns by assuming that such columns can be approximated by infinitely long circular cylinders and hence reduced it to a 2-dimensional problem. Similarly, the flow fields around falling ice plates were studied numerically by Pitter et al. (1973), where the hexagonal plates were approximated with thin oblate spheroids of various axis ratios. Ji and Wang (1990, 1991) and Wang and Ji (1997) made further improvements of these earlier studies by using more realistic ice crystal shapes for their flow field studies. They used circular cylinders of finite length to approximate the hexagonal ice columns and exact hexagonal plates for ice plates, and solved the relevant Navier–Stokes equations to obtain the flow fields. Their results cover low to intermediate Reynolds number range and include both steady and unsteady flow fields. Wang and Ji (1997) compared their results with laboratory experiments for both 2D and 3D cylinders and showed good agreement between the two, indicating that numerical solutions of the Navier–Stokes equations can successfully simulate the flow fields around falling ice crystals.

Because of the non-spherical symmetry of the ice crystal shape, the flow field changes when an ice crystal falls at a different inclination angle with respect to the air stream. All these previous calculations assumed that ice crystals fall with their largest dimension, viz., the length-axis of the column or the basal plane of the plate, oriented perpendicular to the air flow, which is the usual fall orientation of these crystals under steady fall assumption (e.g., page 421 of Pruppacher and Klett, 1997). However, it is well known that snow crystals do not fall straight but the motion depends on the Reynolds number and dimensionless moment of inertia (Willmarth et al., 1964; Zikmunda and Vali, 1972; Field et al., 1997). Using natural plate-like snow crystals, Kajikawa (1992) showed that the unstable falling motions occur for the size as small as 1.23 mm (Reynolds number of 47). Furthermore, he suggested that the large variation of horizontal velocity plays an important role in the aggregation of the crystals having a similar shape and size.

The quantitative evaluation of the electric field alignment has been challenging due to our lack of knowledge of the hydrodynamical torque acting on the crystals. Based on Stokes flow and potential flow, Weinheimer and Few (1987) concluded that crystals of 200 μm to 1 mm tend to align under the electric field of 100 kV/m. However, as he notes that the torque computed by potential flow overestimates the actual one, more accurate estimates of the torque are necessary for intermediate Reynolds number flow. The orientation model of Klett (1995) indicates that the average tilt angles for a small column and plate can be more than 30° due to the Brownian motion, but the angle rapidly decreases to 0 after reaching the size of 40 μm. It further shows that only strong turbulence can affect the average title angles of the larger crystals. Weinheimer and Few (1987) and Klett (1995) both recognize that the hydrodynamic torque distributions were not available at that time for Reynolds number of O(1) and larger. The characteristics of the flow fields around such inclined crystals and their impact on cloud microphysical processes have never been systematically studied before.

The present study represents our attempt to understand the flow characteristics and hydrodynamic torque of the inclined falling ice crystals. We numerically solve the relevant Navier–Stokes equations for a set of inclined ice columns and plates falling in air to obtain the flow fields. In this paper, differences in the flow fields between the stable and inclined orientation of falling are described for low and intermediate Reynolds numbers. Then, we discuss the torque characteristics and its application to electrical alignment of crystals. The list of symbols is given in Appendix D.

2. Physics and mathematics of the problem

In this study, we consider the incompressible flow of air past an ice crystal whose center is fixed at the origin but whose axis along the largest dimension forms an inclination angle θ to the horizontal x-axis. Figs. 1 and 2 show the schematic of this configuration. For an ice column the largest dimension is its length (along its crystallographic c-axis) while for an ice plate the largest dimension is the width of its basal plane (along its crystallographic a-axis) (See Pruppacher and Klett, 1997, Chap. 3). Note that the inclination is oriented along the x–z plane (see Figs. 1b and 2b).

Under the steady-state incompressible condition, the momentum equation (Navier–Stokes equation) and continuity equation are written as (see Pruppacher and Klett, 1997, Chapter 10):

$$\frac{\partial u_j u_i}{\partial x_j} = -\frac{1}{\rho_a}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_j}{\partial x_j{}^2} (i = 1,\ 2,\ \text{and}\ 3) \tag{1}$$

$$\frac{\partial u_j}{\partial x_j} = 0, \tag{2}$$

where u_i is the air velocity ($i = 1, 2,$ and 3 corresponding to x, y, and z axes), p is the air pressure, ρ_a is the air density, and ν is the kinematic viscosity of air. The boundary conditions are:

$$u_i = 0\ \ (i = 1,\ 2,\ \text{and}\ 3)\ \text{at the surface of the crystal} \tag{3}$$

$$u_3 = u_\infty\ \text{at the inlet} \tag{4}$$

$$\frac{\partial u_3}{\partial x_3} = 0\ \text{at the outlet} \tag{5}$$