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The horizontal space–time scaling and cascade structure of the atmosphere and satellite radiances

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ABSTRACT

Classically, turbulence has been modeled by a hierarchy of different isotropic scaling regimes. However, gravity acts at all scales and theory and modern observations point towards an atmosphere described by a single anisotropic scaling regime with different scaling laws in the horizontal and vertical directions: the 23/9D model. However, the implications of this anisotropic spatial scaling for the temporal statistics (i.e. the full space–time scaling) have not been worked out and are the subject of this paper. Small structures are advected by larger turbulent structures, by considering averages over the latter we obtain estimates for the structure functions and spectra.

To test these predictions, we analyze geostationary satellite MTSAT Infra red radiances over wide scale ranges in both horizontal space and in time (5 km to ~10000 km, 1 h to 2 months). We find that our model accurately reproduces the full 3D (k_x, k_y, ω) spectral density up to 5000 km in space and 100 h in time. For example, to within constant factors, the 1D spectral exponents were the same in both horizontal directions and in time with spectral exponent β ~1.55 ± 0.01. We also considered the various 2-D subspaces ((k_x, k_y), (k_x, ω), (k_y, ω)) and showed how these could be used to determine both mean advection vectors (useful for atmospheric motion vectors) but also the turbulent winds.

Going beyond these second order statistics we tested the predictions of multiplicative cascade models by estimating turbulent fluxes from both MTSAT but also the polar orbiting TRMM satellite at infrared and passive microwave bands over scale ranges 100 km to 20000 km, 1 day to 1 year. These accurately obeyed the predictions of multiplicative cascade models over large ranges of spatial scales with typically slight deviations at smallest and largest scales. Analogous temporal analyses showed similar agreement at small scales, but with significant deviations at scales larger than a few days, marking two regimes, associated with weather and macroweather. This allows us to determine Eulerian frame space–time diagrams relating the sizes and lifetimes of structures.

1. Introduction

Turbulent flows have long been recognized for their complexity, randomness and myriad of structures of different sizes and lifetimes. Typically, one describes the statistics of the corresponding fluctuations with the help of scaling laws. For instance, the celebrated Kolmogorov law (Kolmogorov, 1941) describes how turbulent wind fluctuations change with scale. In real space, this law has the form: $\Delta v = \varepsilon^{1/3}\Delta x^H$ where Δv is a fluctuation in the turbulent wind field v, Δx is the spatial separation over which Δv is calculated, H is the mean fluctuation scaling exponent and ε is the flux of energy from large to small scales. The Kolmogorov law applies to statistically isotropic turbulence in three spatial dimensions and the dimensional arguments based on a homogeneous energy flux from large to small scales yield $H = 1/3$. We can also express these laws in Fourier space where they follow $E(k) = \varepsilon^{2/3}\, k^{-\beta}$ where $E(k)$ is

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the power spectrum of the turbulent field, k is the wavenumber and the spectral exponent $\beta = 1 + 2H$, hence the famous "5/3 law".

If we apply the Kolmogorov law to the atmosphere, we must understand and account for the gravity-induced stratification. In the classical quasi-geostrophic approach (Charney, 1971), this is handled by considering the stratification to occur at the very largest scales which are modeled by (quasi flat) horizontal isotropic (2D) turbulence with the smallest scales being isotropic in three dimensions. However gravity acts at all scales, not only the largest and empirical evidence points towards a spatially anisotropic scaling atmosphere (see the reviews (Lovejoy and Schertzer, 2010, 2013)). In the 23/9D model (Schertzer and Lovejoy, 1985b), the turbulence is never isotropic so that (even ignoring intermittency) the classical (isotropic) Kolmogorov law never holds. Nevertheless, the energy flux governs the horizontal dynamics so that the Kolmogorov exponent is still fundamental for the horizontal statistics, and this from the small dissipation scale (~1 mm) up to the largest, planetary scales. In contrast, in the vertical, the buoyancy variance flux dominates the dynamics so that a different exponent is fundamental, the Bolgiano–Obukhov exponent: $H_v = 3/5$ (ignoring intermittency, $\beta_v = 11/5$; Bolgiano, 1959; Obukhov, 1959). The simultaneous action of the two anisotropic cascades leads to an overall 23/9D turbulence model intermediate between flat (2D) and isotropic (3D) turbulence (Schertzer and Lovejoy, 1985a). This model thus has a single anisotropic scaling regime describing the stratification of turbulent structures in the atmosphere, from millimeters to planetary scales.

This 23/9D model provoked a debate sparked by the reinterpretation of aircraft measurements (Lovejoy et al., 2009a, 2009b, 2009c, 2010; Lindborg et al., 2009, 2010; Schertzer et al., 2011; Yano, 2009; Frehlich and Sharman, 2010) followed by the massive (Pinel et al., 2012) re-evaluation of commercial aircraft measurements, and the derivation (Schertzer et al., 2012) of fractional vorticity equations respecting anisotropic scaling symmetries. The latter provide respectively empirical and theoretical arguments in favor of the 23/9D model that are difficult to refute. This raises the question: if the spatial structures do indeed respect anisotropic scaling, what are the implications for the temporal evolution, i.e. the full space–time scaling? Since (x, y, z, t) data sets spanning significant ranges of scales are not available we consider the simpler problem: what are the horizontal–temporal statistics?

Although our goal is to understand Eulerian (fixed frame) statistics, first recall that a general feature of turbulent flows is that there exists a statistical relation between the shears of structures and their lifetimes (their "eddy turnover time"); for Kolmogorov turbulence this is $\Delta v = \varepsilon^{1/2}\Delta t^{H_\tau}$ with $H_\tau = 1/2$, a Lagrangian relation which is used conceptually in meteorology in constructing space–time "Stommel" diagrams (see e.g. Dias et al., 2012). In this paper, instead we estimate the corresponding Eulerian space–time relationships. A classical way to obtain Eulerian statistics is to consider the case when the turbulent fluctuations are sufficiently small compared to an imposed mean flow, such that a clear scale separation exists. Taylor's hypothesis of "frozen turbulence" developed for wind tunnels experiments (Taylor, 1938) can then be used. In this case, a constant (mean flow) velocity V relates temporal to spatial statistics so that $\Delta v = \varepsilon^{1/3}(V\Delta t)^{1/3}$so that $H_\tau = 1/3$ (or β_τ =5/3). However, in the atmosphere, we have argued that no scale separation exists so that another model for space–time scaling is needed.

Without a scale separation, Tennekes (1975) argued that in the Eulerian framework, the turbulent eddies would "sweep" the small eddies. Since the velocity difference across an eddy is ~Δv ~ $\varepsilon^{1/3}\Delta x^{1/3}$, the largest eddies with largest velocities V_e would dominate so that at a fixed location, for time interval Δt, we would have $\Delta v \sim \varepsilon^{1/3}(V_e\Delta t)^{1/3}$ and thus $H_\tau = 1/3$ so that the Eulerian exponent would be different from the Lagrangian one. Radkevich et al. (2008) found empirical support for this by analyzing passive scalar concentrations in the atmosphere (using lidar backscatter as a surrogate), finding values of H_τ mostly $\approx 1/3$ but occasionally $\approx 1/2$. It was argued that the latter values were consequences of the vertical wind dominating the statistics, not a manifestation of the Lagrangian exponent (Lovejoy et al., 2008).

Unfortunately, full (3D) space–time data with wide ranges of scale are not available and reanalyses have limitations, including the use of the hydrostatic approximation (see Stolle et al., 2010; Stolle et al., 2012). Therefore, to better understand the horizontal Eulerian statistics, we present a spectral study of the space–time scaling of atmospheric variability and its (horizontal) space–time statistical relations, using infrared radiances measured by the geostationary multi-functional transport satellite (MTSAT). These infrared radiances are probably the best data currently available for this task as they cover wide scale ranges in both space and time (5 km to ~10000 km, 1 h to months, years). For scenes extracted not too far from the equator, the map projections are straightforward. Here, they do not lead to significant spectral distortions (see the appendix in Lovejoy and Schertzer, 2011). The use of either passive thermal emission bands or active sensing is necessary to avoid strong diurnal effects. However, planetary scale active sensors (satellite-borne radars and lidars) have low temporal resolutions with return times of days. We therefore primarily consider thermal IR from a geostationary satellite which is the best available for the purpose (MTSAT). However, we also analyzed infrared and passive microwave radiances measured by the tropical rainfall measuring mission (TRMM) satellite whose sampling protocol is not ideal for temporal analysis but still allows us to investigate the intermittency.

We should make it clear from the outset that the satellite radiances are not considered as surrogates for cloud liquid water content or any other field. Instead, we use turbulence theory and scaling arguments to derive the corresponding space–time radiance statistics directly (including their space–time spectra). This theoretical form is then empirically tested.

Since our geostationary data is (of necessity) centered at the equator and the sector we analyzed (30°S to 40°N) was largely tropical in character, we will say a few words about the classical (deterministic) dynamical meteorology approach to the tropics. Dynamical meteorology starts with a scale analysis of the governing equations, which is quite different from a scaling analysis (see below), attempting to identify terms which are dominant over a given scale range (usually the so-called synoptic scales). It then considers various idealized flows governed by these dominant terms (e.g. wave motions usually obtained by various linearizations). In this framework the main difference between the tropics and the midlatitudes is the relative lack of Coriolis forces in the former contrasting with