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Analysis of cooperation gain for adaptive networks in different communication scenarios

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ABSTRACT

Adaptive networks are well suited to perform distributed estimation task where the observations collected by nodes of a network are used to estimate a common network-wide desired parameter. Although the benefits of cooperation are clear for the networks with ideal links, in the presence of noisy links their behavior changes considerably. Thus, it is important to analyze, in this case, when and to what extent the cooperation improves the estimation performance. In this paper, we study the influence of noisy links on the effectiveness of cooperation for two important types of adaptive networks, i.e. incremental LMS adaptive network (ILMS) and diffusion LMS (DLMS). We first define the concept of cooperation gain and compute it for the ILMS and the DLMS algorithms with ideal and noisy links. We show that when the links are ideal, both ILMS and DLMS algorithms provide increased performance in comparison with the non-cooperative solution. On the other hand, in the presence of noisy links, the cooperation gain is not always influential, and based on the channel and data statistics, for some values of step-size, a non-cooperative scheme outperforms the ILMS and DLMS algorithms. We present some simulation results to clarify the discussions.

1. Introduction

Distributed in-network signal processing algorithms are receiving significant attention in recent years because of their clear advantage of being cost-effective and easy to deploy compared to the centralized solutions [1,2]. Among the available distributed estimation algorithms, adaptive networks are currently forming an active area of research due to their appealing properties. An adaptive network is a collection of spatially distributed nodes that interact with each other and function as a single adaptive entity that is able to respond to data in real-time and also track variations in their statistical properties [3–5]. Adaptive networks were initially proposed to perform decentralized information processing task [6]. However, the adaptation and learning ability has made it possible for the adaptive networks to model forms of complex behavior exhibited by biological and social networks [7–10]. Based on the mode of cooperation between nodes, adaptive networks can be roughly classified into incremental [3,4,11–17] and diffusion [18–24] algorithms. In incremental based adaptive networks, a Hamiltonian cycle is established through the nodes and each node cooperates only with one adjacent node to exploit the spatial dimension, whilst performing local computations in the time domain. This approach reduces communications among the nodes and improves the network autonomy as compared to a centralized solution. In the diffusion based adaptive networks, on the other hand, nodes communicate with all of their neighbors, and no cyclic path is required.

In this work we consider both incremental and diffusion based adaptive networks, in the context of distributed estimation problem, where observations collected by nodes in the network are use to estimate a common network-wide desired parameter. Such a distributed estimation problem arises in a wide range of applications ranging from precision agriculture to environmental monitoring, military surveillance, and the modeling of self-organization in biological networks [4,11]. We assume a linear regression measurement model and use a least mean squares (LMS) criterion in the estimation procedure because of its simplicity and wide applicability.

Remark 1. It must be noted that in many applications, e.g. vibration monitoring, wireless acoustic sensor networks, and noise reduction in hearing aids, we need to solve a distributed

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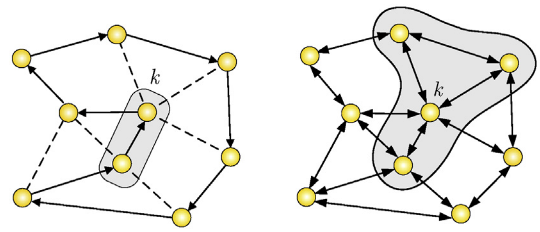

Fig. 1. Different types of cooperation in adaptive networks: incremental (left) and diffusion (right). The nodes in the shaded area denote the neighbors of node k in every cooperation mode.

estimation problem where each node in the network estimates a different node-specific desired parameter [25–27]. This work is different from those in the referred papers since we consider a common, network-wide desired parameter.

Unlike the existing works in [3,4,11–14,16,18–24], in this work we do not assume the links between the nodes to be ideal. This paper extends the results of [28–34] and provides some insights into the steady-state performance of LMS-based adaptive networks with noisy links. Firstly, we define the concept of cooperation gain in adaptive networks which is the MSD achieved by non-cooperative solution, divided by the MSD achieved by the adaptive network. We show that when the connecting links are ideal, for both of incremental LMS (ILMS) and diffusion LMS (DLMS) adaptive networks, the cooperation equals the number of nodes in the network. This means that in this case, cooperation between the nodes in the adaptive network leads always to smaller steady-state error than in a non-cooperative solution and increasing the number of nodes improves the estimation performance. On the other hand, when the links are noisy, the cooperation gain in an adaptive network may be bigger or smaller than 1. In the presence of noisy links, the cooperation gain is a function of the data and channel statistics. Since the cooperation between nodes is not always useful, we need to adjust the learning parameters of adaptive network to ensure that the cooperation beneficial. We undertake some simulations to demonstrate the performance.

The outline of the paper is as follows. In Section 2, we describe the estimation problem and introduce its non-cooperative solution. In Section 3 we review the incremental LMS and diffusion LMS adaptive networks for a distributed estimation problem and present some theoretical expression which demonstrates their steady-state performance in the presence of noisy links. The cooperation gain analysis of adaptive networks is presented in Section 4. The simulation results appear in Section 5. Finally, some concluding remarks are made in Section 6.

Notation: Throughout the paper, we use boldface letters for matrices and vectors and small letters for scalars. We use $\otimes$ to denote Kronecker product and $\odot$ to denote block Kronecker product of two block matrices. The notation * is used to denote complex conjugation for scalars and complex-conjugate transposition for matrices. We also write $\mathbf{B} = \text{bvec}\{\mathcal{B}\}$ to denote the conversion of block matrix $\mathcal{B}$ into a single column. Finally, $\|\mathbf{x}\|^2_{\boldsymbol{\Sigma}}$ represents the weighted norm notation defined as $\mathbf{x}^*\boldsymbol{\Sigma}\mathbf{x}$.

2. The estimation problem and its solutions

Let's consider a set of nodes $\mathcal{N} = \{1, 2, \ldots, N\}$ that are distributed in space. Each node $k \in \mathcal{N}$ communicates with its neighbors (denoted by $\mathcal{N}_k$) according to the cooperation mode. The set of neighbors for node k depends on the cooperation mode of adaptive network (see Fig. 1). At time instant i, node k collects the scalar target $d_k(i)$ and $1 \times M$ regression data $\mathbf{u}_{k,i}$ where in general both $\{d_k(i), \mathbf{u}_{k,i}\}$ can be complexed-valued signals. It is assumed that the $\{d_k(i), \mathbf{u}_{k,i}\}$ are related according to a linear model

$$d_k(i) = \mathbf{u}_{k,i}\mathbf{w}^o + v_k(i) \tag{1}$$

where the $M \times 1$ vector $\mathbf{w}^o$ denotes the unknown parameter of interest, and $v_k(i)$ represents the observation noise term with variance $\sigma^2_{v,k}$. Based on the intended application, the vector $\mathbf{w}^o$ may represent different physical quantities, e.g. location of a target and parameter of an auto-regressive (AR) model. The purpose of the network is to solve the problem of estimating $\mathbf{w}^o$, at every node, using the data collected from the entire network. We can write the problem of estimating $\mathbf{w}^o$ as the following optimization problem

$$\mathbf{w}^o = \min_{\mathbf{w}} \sum_{k=1}^{N} \mathbb{E}|d_k(i) - \mathbf{u}_{k,i}\mathbf{w}|^2 \tag{2}$$

It is straightforward to show that the optimal solution of (2), i.e. $\mathbf{w}^o$ is related to the statistics of collected data $\{d_k(i), \mathbf{u}_{k,i}\}$ as in the following normal equation [4,11]:

$$\left(\sum_{k=1}^{N} \mathbf{R}_{u,k}\right)\mathbf{w}^o = \sum_{k=1}^{N} \mathbf{p}_{du,k} \tag{3}$$

where

$$\mathbf{R}_{u,k} = \mathbb{E}[\mathbf{u}^*_{k,i}\mathbf{u}_{k,i}], \qquad \mathbf{p}_{du,k} = \mathbb{E}[d_k(i)\mathbf{u}^*_{k,i}] \tag{4}$$

It is obvious that every node in the network can use its local data $\{d_k(i), \mathbf{u}_{k,i}\}$ to produce a local estimate of $\mathbf{w}^o$. Let denote by $\mathbf{w}_{k,i}$ the local estimate of $\mathbf{w}^o$ at node k in time i. In a non-cooperative scheme, (i.e. when $\mathcal{N}_k(i) = \{k\}$) each node uses its past estimate to update its local estimate. Using the LMS adaptive algorithms at the node level, the update equation for this solution is given by

$$\hat{\mathbf{w}}_{k,i} = \hat{\mathbf{w}}_{k,i-1} + \mu_k\mathbf{u}^*_{k,i}(d_k(i) - \mathbf{u}_{k,i}\hat{\mathbf{w}}_{k,i-1}), \quad i \geq 0 \tag{5}$$

starting from initial condition $\boldsymbol{\psi}_k^{(-1)} = \mathbf{0}$. In (5) $\mu_k \in \mathcal{R}^{\text{nc}}$ denotes the step size parameter ($\mathcal{R}^{\text{nc}} \subseteq \mathbb{R}^{>0}$ is the convergence region for LMS algorithm [35]). The performance of the non-cooperative solution (5) is well studied in the literature [35,5]. A good measure to evaluate the steady-state performance of non-cooperative solution is the MSD, where for non-cooperative scheme (5), is defined as

$$MSD_k^{\text{nc}} \triangleq \lim_{i\to\infty} \mathbb{E}[\|\tilde{\mathbf{w}}_{k,i}\|^2_{\mathbf{I}}] \tag{6}$$

where

$$\tilde{\mathbf{w}}_{k,i} \triangleq \mathbf{w}^o - \hat{\mathbf{w}}_{k,i} \tag{7}$$

We added superscript nc to emphasis that this result is for non-cooperation solution. It is shown in [35] that for the non-cooperative scheme (5), the MSD is given by

$$MSD_k^{\text{nc}} = \frac{\mu M}{2}\sigma^2_{v,k} \tag{8}$$

The average MSD over all nodes of network is given as

$$\overline{MSD}^{\text{nc}} = \frac{\mu M}{2}\left(\frac{1}{N}\sum_{k=1}^{N}\sigma^2_{v,k}\right) \tag{9}$$

Although the non-cooperative scheme (5) can estimate the unknown vector $\mathbf{w}^o$, but, it is a well-known fact that the distribution of the nodes in an area yields spatial diversity, which can be exploited alongside the temporal dimension to improve the estimation performance [36]. In the sequel, we briefly introduce two different distributed estimation algorithms which are able to solve the linear estimation problem (2) in a fully cooperative fashion, in which nodes equipped with local computing abilities.