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Simulation of pure sedimentation of raindrops using quadrature method of moments

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ABSTRACT

The quadrature method of moments (QMOM) is used for simulation of pure sedimentation of raindrops in a one-dimensional rainshaft. The moments have been calculated in three ways, based either on droplet diameter using two and three nodes or on droplet volume using two nodes. The method gives useful information on the range of sizes and also on spatial segregation of the droplets. The results show that all three methods give useful information about the transport processes involved and compare satisfactorily with the spectral (bin) method although use of diameter as internal coordinate is preferable from the viewpoint of quantitative agreement.

1. Introduction

Simulation of the evolution, in particular sedimentation, of an ensemble (a "population") of raindrops has been attempted by numerous researchers over the last decades. The motivation for continued research in this area is to develop microphysical models that capture the essential physics, but are simple and efficient enough to be incorporated in three-dimensional meteorological simulations. Generally, the efforts for finding the drop size distribution (DSD) can be categorized into two broad classes (Khain et al., 2000): "discrete methods", also referred to in literature as "method of classes", "bin method" or "spectral methods" (SM) and "method of moments" (MOM). In the first category (Hall, 1980; Kogan, 1991; Bott, 1997; Prat and Barros, 2007), the range of drop sizes is divided into a finite number of size classes and transport equations are solved for each size class. This method is conceptually straightforward, as the polydispersity is directly addressed. But for accurate simulations, discretization into a large number of classes is necessary, which makes the approach too expensive for 3-D simulations (Seifert and Beheng, 2006). On the other hand, in the method of moments (Enukashvili, 1964; Tzivion et al., 1987; Seifert and Beheng, 2006), the transport equations for statistical moments of the size distribution are solved. Each moment is generally transported with a distinct velocity different from those of both continuous and discrete phases. Another difficulty with the method of moments is that the source terms in the transport equations generally involve moments other than those being solved and integrals over the particle size, which are difficult to express in terms of moments. This calls for additional "closure relations". A variety of approaches for this closure have been proposed in the literature. The traditional closure method involves assumption of a form for the size distribution function. Tzivion et al. (1987) and Feingold et al. (1988) divided the entire range of sizes into a finite number of classes and defined moments for each section. They closed the moment problem by relating higher order moments to two lower order moments and assuming linear and cubic functions to describe the size distribution in each size interval. The early work of Kessler (1969) and some subsequent works (Lin et al., 1983; Walko et al., 1995) used a one-moment description conserving only the liquid mass. However, as shown by Beheng and Doms (1986), mass conservation alone is inadequate for correct representation of droplet size distribution in presence of breakup, coalescence and other effects like autoconversion

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and accretion. The two moment method, conserving liquid mass and droplet number density, adopted by Tzivion et al. (1987), Seifert and Beheng (2000), Meyers et al. (1997) and Lüpkes et al. (1989), performs better in presence of the above effects. Many authors (e.g., Kessler, 1969; Mcfarquhar and List, 1991 and Wacker and Seifert, 2001) used the Marshall–Palmer distribution, originally proposed by Marshall and Palmer (1948) in their model for sedimentation of raindrops using one moment and two moment models. Prat and Barros (2007) also used this distribution as the initial condition. Recently, Wacker and Lüpkes (2009) addressed the problem of pure sedimentation of raindrops by method of moments using one and two moment schemes. They compared their solutions for different choice of moments (for which conservation equations were solved) with spectral solution as reference. They assumed gamma function as the form of the distribution function. Milbrandt and Yau (2005a, 2005b) introduced a three moment scheme to predict the segregation of droplets in a pure sedimentation process. They observed the three moment model to have better quantitative agreement with an analytical bin model than the two moment models.

The method of moments has been used in other fields of application, like aerosol dynamics (McGraw, 1997), crystal growth and aggregation (Marchisio et al., 2003), bubble dynamics (Carneiro et al., 2008) and droplet dispersion and evaporation in a spray (Carneiro et al., 2009). Often researchers in meteorology have adopted solution strategies from those of similar problems in these other fields. For example, Prat and Barros (2007) extended the "fixed pivot technique" used by Kumar and Ramkrishna (1996) for solution of population balance equations to include the phenomenon of collisional breakup. In many of the above works (e.g., McGraw, 1997; Marchisio et al., 2003) the closure of the moments is achieved using a technique known as "quadrature method of moments" (QMOM). A major motivation for using QMOM in simulating polydisperse flows is that the method does not require any prior knowledge of the size distribution function. However, QMOM has not been applied to meteorological problems to the best knowledge of the authors. The method has been shown to perform very well (Drumm et al., 2009) in presence of complicated source terms representing phenomena like coalescence and breakup of particles.

In the meteorological literature, droplet size is often expressed in terms of either mass (or equivalently volume for incompressible liquid) or diameter distribution functions though some models use both mass and diameter selectively for different species. In contrast, in other applications like bubble dynamics and crystal growth, the size distribution is generally expressed as a function of particle diameter. The implications of the two alternative approaches with regard to accuracy and computational cost are not readily apparent. Moreover, correlation of the moments based on diameter and volume is also not trivial.

The objective of the present work is to implement QMOM in a meteorological problem. As a first step, pure sedimentation of raindrops in a one-dimensional rainshaft is simulated to illustrate the information provided by this solution technique and to examine its effectiveness in addressing microscale issues of rainfall simulation. In spite of its simplicity, the problem addresses complexities like non-equal transport velocities for different moments resulting from size-dependent particle velocities. In the majority of the works dealing with QMOM, the issue of non-equal moment transport velocities has not been addressed. Transport equations for moments based on both diameter and volume are solved to compare the two alternative approaches.

2. Mathematical model

The microphysical state of an ensemble of raindrops in a one-dimensional rainshaft is described by the size distribution function $n(D,z,t)$, where D represents the diameter of a drop at height z and time t. The number of drops per unit volume in the diameter interval $(D, D + \mathrm{d}\,D)$ is given by $n(D,z,t)\mathrm{d}\,D$. Since in this work, only pure sedimentation is considered in a one-dimensional "rainshaft", effects of collision, coalescence, breakup condensation and advection are neglected. Thus the formulation of the problem is similar to that of Wacker and Seifert (2001). The terminal velocity of the individual drops depends on the drop diameter. The magnitude of the drop velocity is related to its diameter through a power-law relation:

$$u(D) = \alpha\left(\frac{D}{D_v}\right)^{\beta} \tag{1}$$

Following Wacker and Seifert (2001), the following values recommended by Kessler (1969) are adopted for the parameters in Eq. (1): $\alpha = 13$ m/s, $D_v = 0.01$ m and $\beta = 0.5$. The transport equation is given by

$$\frac{\partial n(D,z,t)}{\partial t} - \frac{\partial}{\partial z}[u(D)n(D,z,t)] = 0 \tag{2}$$

In the above equation, the negative sign indicates the fact that the positive z coordinate points vertically upward, while the terminal velocity is directed vertically downward.

2.1. Transport equations for moments based on particle diameter

The transport equations for different moments of the above distribution are obtained by multiplying Eq. (2) with D^k and integrating the product over the entire range of diameters. Thus

$$\frac{\partial}{\partial t}\int_0^{\infty} D^k n(D,z,t)dD - \frac{\partial}{\partial z}\left[\int_0^{\infty} D^k u(D)n(D,z,t)dD\right] = 0 \tag{3}$$

In Eq. (3), the order of the integral and differential operators has been interchanged assuming smoothness of the function. The moments of the distribution and their transport velocities are defined as

$$M^{(k)} = \int_0^{\infty} D^k n(D)dD, \tag{4a}$$

$$u^{(k)} = \frac{1}{M^{(k)}}\int_0^{\infty} D^k n(D)u(D)dD \tag{4b}$$

Using these definitions, the moment transport equations become

$$\frac{\partial M^{(k)}}{\partial t} - \frac{\partial}{\partial z}\left(u^{(k)}M^{(k)}\right) = 0 \tag{5}$$