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On the theory of intensity distributions of tornadoes and other low pressure systems

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ABSTRACT

Approaching from a theoretical point of view, this work presents a theory which unifies intensity distributions of different low pressure systems, based on an energy of displacement. Resulting from a generalized Boltzmann distribution, the expression of this energy of displacement is obtained by radial integration over the forces which are in balance with the pressure gradient force in the horizontal equation of motion. A scale analysis helps to find out which balance of forces prevail. According to the prevailing balances, the expression of the energy of displacement differs for various depressions. Investigating the system at the moment of maximum intensity, the energy of displacement can be interpreted as the work that has to be done to generate and finally eliminate the pressure anomaly, respectively. By choosing the appropriate balance of forces, number–intensity (energy of displacement) distributions show exponential behavior with the same decay rate β for tornadoes and cyclones, if tropical and extra-tropical cyclones are investigated together. The decay rate is related to a characteristic (universal) scale of the energy of displacement which has approximately the value $E_u = \beta^{-1} \approx 1000\ \mathrm{m^2 s^{-2}}$. In consequence, while the different balances of forces cause the scales of velocity, the energy of displacement scale seems to be universal for all low pressure systems. Additionally, if intensity is expressed as lifetime minimum pressure, the number–intensity (pressure) distributions should be power law distributed. Moreover, this work points out that the choice of the physical quantity which represents the intensity is important concerning the behavior of intensity distributions. Various expressions of the intensity like velocity, kinetic energy, energy of displacement and pressure are possible, but lead to different behavior of the distributions.

1. Introduction

An accurate description of statistical intensity distributions of tornadoes and cyclones, especially extreme ones, is of great interest for risk assessments of the future occurrence of extreme events as well as for climatological aspects and modeling. Tornadoes have been investigated in detail concerning their number–intensity distributions (e.g. Brooks and Doswell, 2001; Kurgansky, 2000; Dotzek et al., 2003; Feuerstein et al., 2005; Dotzek et al., 2005). The intensity of tornadoes is mostly given in Fujita's intensity scale, which has been introduced by Fujita (1971), who classified tornadoes in 6 classes by their degree of caused damage. Fujita connected each damage class threshold to a wind speed by $v(F) = 6.30\ \mathrm{ms^{-1}}(F+2)^{3/2}$ where $F = \{0,1,2,3,4,5\}$ is the class number. Brooks and Doswell (2001) analyzed worldwide tornado data and concluded that the intensity distributions of tornadoes in regions where violent tornadoes of F4 and F5 occur are approximately exponentially distributed. US tornado data distributions over several decades even showed similar slopes for tornadoes of Fujita intensity classes F2 to F4. Further investigations showed, that worldwide tornado distributions including all Fujita intensity classes are best approximated with help of Weibull statistics in Fujita wind speed v_F as well as in Fujita-classes F (Dotzek et al., 2003; Feuerstein et al., 2005). As a special case of Weibull statistics with form parameter $c = 2$ tornado

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distributions can be approximated by Rayleigh distributions in Fujita wind speed v_F (Kurgansky, 2000; Dotzek et al., 2005). A Rayleigh distribution concerning wind speed is connected to an exponential distribution concerning mass-specific kinetic energy $v_F^2/2$.

Golitsyn et al. (2007) analyzed cumulative number–intensity distributions of untracked extra-tropical cyclone and anticyclone data based on NCEP/NCAR Reanalysis in a period of 49 years (1952–2000). They found out that the cumulative distributions of cyclones within a range of pressure deviations between the center and the edge of 4.5 hPa $\leq \delta p \leq$ 25 hPa show approximately exponential behavior concerning their kinetic energy and their area, although the exponential parameters differ. Akperov et al. (2007) used the same method to expand the analysis concerning climate change scenarios. Moreover, frequency–intensity distributions of extra-tropical cyclones are used to compare different data sets. Hanson et al. (2004) used frequency distributions to compare data obtained by different data sets. Pinto et al. (2005) used frequency–intensity distributions to compare the frequency of cyclones in the Mediterranean Sea and North Atlantic. In addition, the intensity of extra-tropical cyclones can be given by minimum central pressure (e.g. Schinke, 1993) and minimum geopotential height respectively, maximum core vorticity which is related to the Laplacian of the pressure (e.g. Pinto et al., 2005), or Beaufort intensity scale (Hanson et al., 2004), which is related to wind speed by $v(B) = 0.835\ \mathrm{ms}^{-1} B^{3/2}$ with $B = \{1,\ldots,12\}$ (after Müller, 1979) representing the Beaufort intensity class.

The intensity of tropical depressions, tropical storms, hurricanes, and typhoons respectively (herein after all these systems are referred to as tropical cyclones) is given by the measurements of the maximum of the 1 min averaged wind speed in 10 m above ground level or by the measured minimum central pressure (e.g. Landsea, 1993). Additionally, north Atlantic and eastern north Pacific tropical cyclones are classified by the Saffir-Simpson intensity scale (cf. Simpson, 1974). which ranges from 1 to 5 starting with hurricane wind speed given by an 1 min average of 33 ms^{-1}. A similar classification is used in the western north Pacific with the term typhoon instead of hurricane. Cumulative frequency–intensity distribution functions (CDFs) of tropical cyclones has been analyzed by Emanuel (2000), who used a normalized wind speed as intensity. The normalization is defined as the ratio of lifetime maximum wind speed and the potential wind speed at the same place and time. The CDFs were nearly linear for hurricanes and tropical storms with different slopes, indicating that any given tropical cyclone has a uniform probability to attain any intensity up to hurricane wind speed, and a uniform but lower probability that it will achieve any intensity between hurricane force and its potential intensity. Emanuel (2000) supposes that any climatic change in potential intensity would affect the distribution uniformly.

Various expressions of intensity are possible for investigating number–intensity distributions of tornadoes, extra-tropical lows, and tropical cyclones as has been demonstrated in the text above. The here presented work proposes an expression that is obtained from a theoretical point of view giving the advantage of a unified view on these different low pressure systems. The common baseline is an energy of displacement which is obtained by the integration over the forces that balance the pressure gradient force in the radial component of the horizontal equation of motion. In case of maximum intensity, the energy of displacement describes the work that has to be done to generate the low pressure system and that is released during the refilling process. Therefore, the horizontal equation of motion will be investigated by scale analysis and approximated concerning the prevailing horizontal balance of forces. According to the prevailing balances, the expression of the energy of displacement differs for various low pressure systems. Using the appropriate expression of this energy of displacement as intensity, empirical number–intensity (energy of displacement) distributions show exponential behavior for all depressions. Moreover, number–intensity (pressure) distributions should be power law distributed.

In Section 2 the theory is presented in general. The prevailing balances of tornadoes, extra-tropical cyclones, and tropical cyclones and their integrations is treated in detail in Section 3. In Section 4, the used data sets are described. The results concerning number-intensity (energy of displacement) are presented in Section 5. An extension of the theory leading to power law behavior of number–intensity distributions concerning pressure and a discussion of the different interpretations of exponential and power laws is given in Section 6. Finally, a conclusion of this work can be found in Section 7.

2. Theory

The equation of motion in natural coordinates splits into two components: one is normally directed, the other one is streamwise directed. If friction is neglected, the streamwise component is given by the balance of absolute value of the acceleration following the motion and streamwise directed pressure gradient: $dv/dt = -\rho^{-1}\partial p/\partial s$. The normal component of the equation of motion describes the balance of the (mass-specific) forces. The normally directed pressure gradient force is in balance with the sum of centrifugal force and Coriolis force (friction is neglected):

$$\frac{v^2(r)}{r} + fv(r) = F(r) = -\frac{1}{\rho}\frac{\partial p}{\partial n}. \tag{1}$$

Here, $v(r)$ represents the horizontal velocity, depending on radius r and f is the Coriolis parameter. The sum of mass-specific centrifugal force and Coriolis force is denoted as $F(r)$. Concerning an idealized circularly-shaped low pressure anomaly at the moment of maximum intensity the acceleration and therefore the streamwise component become zero. In consequence, the pressure field is only changing in radial direction ($\partial p/\partial n = -dp/dr$). Using the ideal gas law $p = \rho RT$, Eq. (1), becomes $dp/p = F(r)/(RT)dr$. Integrating over the radius of a general circularly-shaped pressure system assuming a constant temperature T leads to a generalized Boltzmann distribution:

$$P(E) = P_0 exp(-E / RT), \tag{2}$$

where P is the pressure at the center of the low pressure system and P_0 is the environmental pressure in distance of the radius r_0 of the system. The energy of displacement E is defined as the