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Review article

The state-of-art of the GILTT method to simulate pollutant dispersion in the atmosphere

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A B S T R A C T

In this work, we present a review of the GILTT (Generalized Integral Laplace Transform Technique) solutions for the one and two-dimensional, time-dependent, advection–diffusion equations focusing the application to pollutant dispersion simulation in atmosphere, assuming both Fickian and counter-gradient models for a wide class of problems. For sake of completeness, we also report numerical simulations and statistical comparisons with experimental data and results of literature.

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1. Introduction

In the last years, special attention has been given to the issue of searching analytical solutions for the advection–diffusion equation in order to simulate the pollutant dispersion in the Planetary Boundary Layer (PBL). The solution of the advection–diffusion equation can be written either in integral form and series formulations, with the main property that both solutions are equivalent. At this point we are aware of the existence of many closed-form solutions in the literature. Among them we mention the works of Rounds (1955), Smith (1957), Scriven and Fisher (1975), Demuth (1978), van Ulden (1978), Nieuwstadt and de Haan (1981), Tagliazucca et al. (1985), Tirabassi (1989), Tirabassi and Rizza (1994), Sharan et al. (1996), Lin and Hildemann (1997), Tirabassi (2003). All these solutions are valid for very specialized practical situations with restrictions on wind and eddy diffusivities vertical profiles. Recently appeared the ADMM (Advection Diffusion Multilayer Method) approach (Costa et al., 2006) that solves the multidimensional advection–diffusion equation for more realistic physical scenario. The main idea relies on the discretization of the PBL in a multilayer system, where in each layer the eddy diffusivity and wind profile assume averaged values. The resulting advection–diffusion equation in each layer is then solved by the Laplace Transform technique. For more details about this methodology see the revision work by Moreira et al. (2006a). In this work, we focus our attention to the state-of-the-art now for the series solution, of the advection–diffusion equation, known in the literature as the GILTT (Generalized Integral Laplace Transform Technique) approach. The main idea of this methodology comprehends the steps: expansion of the concentration in series of eigenfunctions attained from an auxiliary problem, replacing this equation in the advection–diffusion equation and taking moments, we come out with a matrix ordinary differential equation that is solved analytically by the Laplace Transform technique.

To reach our objective, we organize the paper as follows: in Section 2, we report the advection–diffusion equation solutions by the GILTT approach. In Section 3, we present the solution for counter-gradient turbulence closure. In Section 4, we report the turbulent parameterizations. In Section 5, we display the wind profile used in the simulations. In Section 6, we display numerical simulations and statistical comparisons with experimental data and results of literature. Finally, in Section 7, we make mathematical analysis and discussion of the results performance attained by this methodology in the conclusions.

2. The advection–diffusion equation and the GILTT method

In the sequel, we report the GILTT solution for the advection–diffusion equation for the problems: one-dimensional time-dependent equation, two-dimensional steady-state equation, two-dimensional steady-state equation with deposition at the ground, two-dimensional time-dependent equation with advection in the vertical direction and two-dimensional time-dependent equation with longitudinal diffusion and vertical velocity.

2.1. The time-dependent one-dimensional advection–diffusion equation

Let us consider the following equation:

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial z}\left(K_z\frac{\partial c}{\partial z}\right), \tag{1}$$

for $0<z<h$ and $t>0$, subject to the boundary conditions of zero flux at the ground and PBL top, and a source with emission Q at height H_s:

$$K_z\frac{\partial c}{\partial z}=0 \quad \text{at } z=0,h \tag{1a}$$

$$c(z,0)=Q\delta(z-H_s) \quad \text{at } t=0 \tag{1b}$$

where c represents the crosswind integrated concentration, h is the PBL height, K_z is the eddy diffusivity variable with the height z ($K_z=K(z)$) and δ is the Dirac delta function. The diffusive term in the Eq. (1) is rewritten using the chain rule. This procedure was used by Wortmann et al. (2005) and allows a simplification of the auxiliary problem, whose choice is made as customary in the use of GITT (Generalized Integral Transform Technique) due to Cotta and Mikhaylov (1997). Then, we can write:

$$\frac{\partial c}{\partial t}=K_z\frac{\partial^2 c}{\partial z^2}+K_z'\frac{\partial c}{\partial z}. \tag{2}$$

The formal application of GITT begins with the choice of the following auxiliary Sturm–Liouville problem:

$$\Psi_n''(z)+\lambda_n^2\Psi_n(z)=0 \quad \text{at } 0<z<h \tag{3a}$$

$$\Psi_n'(z)=0 \quad \text{at } z=0,h, \tag{3b}$$

which has the well-known solution $\Psi_n(z)=\cos(\lambda_n z)$, where $\lambda_n=n\pi/h$ ($n=0,1,2,\ldots$).

Next, we expand the concentration $c(z,t)$ in the truncated series as follows:

$$c(z,t)=\sum_{n=0}^{N}\bar{c}_n(t)\Psi_n(z). \tag{4}$$

To determine the unknown coefficient $\bar{c}_n(t)$ we replace Eq. (4) in Eq. (2). This procedure leads to:

$$\sum_{n=0}^{N}\bar{c}_n'(t)\Psi_n(z)=K_z\sum_{n=0}^{N}\bar{c}_n(t)\Psi_n''(z)+K_z'\sum_{n=0}^{N}\bar{c}_n(t)\Psi_n'(z). \tag{5}$$