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A simple model of rain in time: An alternating renewal process of wet and dry states with a fractional (non-Gaussian) rain intensity

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Abstract

A simple model of rainfall in time is proposed coupling the theory of renewal processes with a scale invariant representation of rain intensity. In particular, a strictly alternating renewal process mimes the sequence of wet and dry synoptic weather states, while a Fractional Noise represents the rain variability within each wet state. The rain model is adapted to Osservatorio Ximeniano (Florence) and Osservatorio di Brera (Milan) datatasets. Some rain characteristics are considered to check the agreement between model and data, namely, the *annual volume of rainfall*, the *wet fraction of the year*, the *extreme values* through the “classic” Depth–Duration–Frequency curves, and the *maximum annual volume of the rainfall event*.

Keywords: Rainfall; Model; Renewal process; Scale invariance; Fractional Noise; Spectral density; Calibration; Validation

1. Introduction

The rainfall is a puzzling phenomenon, and modeling it is yet an open issue. Here, we propose a simple rainfall model which considers both *external and internal structure of rainfall*. The former one, which describes the alternation of wet and dry synoptic weather conditions, is modeled through a simple renewal process. The latter one, which represents the variability of rainfall intensity within the wet period, is modelled through a Fractional non-Gaussian Noise. The paper is organized as follows: in Section 2, a description of the background knowledge in rainfall modelling is given, and related problems are discussed. In Section 3, we briefly recall some useful concepts that will be used later: the temporal scale invariance of rainfall, and the probability distributions of some random variables (RVs), namely, wet period, dry period, and rainfall intensity. In Section 4, we introduce a new simple model of rainfall. In Section 5, we illustrate two case studies comparing some rain characteristics of observed and simulated sequences. In Section 6 the results are discussed.

2. Background knowledge

In Literature, almost the totality of the available rain models is of flux type. These mime the variability of the rain intensity, i.e. the flux of water falled over a fixed space, and in a fixed time. The development of this kind of models was also supported by the instruments used to

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collect the rain (standard rain gauge, weighing rain gauge, or tipping bucket rain gauge).

The first generation of rain models in time, (Le Cam, 1961; Waymire and Gupta, 1981; Rodriguez-Iturbe et al., 1987; Cowpertwait, 1991; Cowpertwait, 1994; Cernesson et al., 1996), still widely used in operational hydrology, is calibrated and used at a fixed temporal resolution (usually not smaller than 1 day), where the rain in time was considered scale dependent (see Zawadzky (1973)). Zawadzky found an exponential decay of the autocorrelation function in time, indicating a scale dependent behavior of rain. In this view, Cowpertwait (1994) proposed a rain model in time characterized by scale dependent statistics. Scale dependent models are still currently used for practical applications, see Arnaud and Lavabre (1999, 2002). Generally, these models are hyperparametrized, e.g., the Arnaud and Lavabre (1999) model has 21 parameters. However, even when the number of parameters is small, these models are not suitable for time resolutions different from the one used to calibrate it.

Recently, the concept of scale invariance has gained consideration in the analysis of rainfall, since it can explain the rainfall variability passing from a scale to another one, and simplify the mathematical tractability of the problem. In Literature, many works provide evidences of temporal scale invariance of rainfall over a range of scales, (see e.g., Olsson et al. (1993), Hubert et al. (1993), Venugopal and Foufoula-Georgiou (1996), Veneziano et al. (1996), Pavlopoulos and Gupta (2003), Venugopal et al. (2006), among others). Even some traditional models built without any scaling assumptions have been shown to hold scaling properties in some range of scales (see e.g., Olsson and Burlando (2002)). Recently, various authors proposed generators of point rainfall characterized by scale invariance statistics. Schmitt et al. (1998) used the theory of multifractals, and cascades coupled with the renewal processes to model the temporal variability of rainfall; Menabde and Sivapalan (2000) considered the theory of the bounded random cascades with Levy-stable distributions of the RVs; Veneziano and Furcolo (2002) proposed a combination of a traditional Poisson process, for the alternation of wet and dry periods, with a hierarchical pulse model to represent the variability of rainfall; Deidda et al. (1999) used the wavelets to reproduce the multifractal properties of the rainfall series. Note that due to the complexity of multifractals, in the operational hydrology the application of these models is sometimes limited. Anyway, the scale invariance (or scale free) behavior of rainfall in time remains an interesting, and challenging, issue in the analysis and simulation of rainfall fields.

The probability distribution of relevant RVs for rainfall modeling represents another important issue. Since Eagleson (1978), RVs like average rainfall intensity, wet period and dry period, that represent the external structure of the rainfall, have been assumed independent with each other, and exponentially distributed. Recently, the hypothesis of exponentiality of these RVs has been discussed at the light of a scaling invariance behavior of rainfall. Indeed a power-law tail behavior is generally predicted by scale invariance models and it arises from physically based multifractal models, such as dressed multiplicative cascades (see e.g., Schertzer and Lovejoy (1987)). Moreover, some evidences of power-law decay of the right tail of the rainfall intensity, and wet and dry periods RVs are found, see for instance, Salvadori and De Michele (2001, 2006), Peters et al. (2002), Peters and Christensen (2002), De Michele and Salvadori (2003), Pavlopoulos and Gupta (2003).

Definitely, a rainfall model should capture the physical properties of the rain, like the scale invariance, reproduce the statistical behavior of the involved RVs, and be simple to improve and easy to manage.

3. Scaling and probability in rain modeling

In Section 3.1 we give some basic notions about the temporal scale invariance of rain, while in Section 3.2 we show the probability distribution used to model the considered RVs: wet period W, dry period D, and rainfall intensity I, and the method used to estimate the parameters of the distribution.

3.1. Temporal scale invariance

The scale invariance of the second order properties of rainfall in time (if they exist) can be easily investigated through the spectral analysis. If a point stochastic process exhibits a scale invariant behavior in time, then its spectral density function, S, scales with the frequency f as a power law

$$S(f) \propto \frac{1}{f^{\beta}}, \tag{1}$$

where β is the scaling exponent. Spectral analysis is widely used to assess the scale invariant behavior of rainfall in time or space–time domain, see e.g., Menabde et al. (1997), Malamud and Turcotte (1999), and De Michele and Bernardara (2005). An estimate of β can be obtained making a simple linear regression of the logarithm of the sample power spectrum vs the logarithm of the frequency.