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Self-similarity in the equation of motion of a ship

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ABSTRACT: *If we want to analyze the motion of a body in fluid, we should use rigid-body dynamics and fluid dynamics together. Even if the rigid-body and fluid dynamics are each self-consistent, there arises the problem of self-similar structure in the equation of motion when the two dynamics are coupled with each other. When the added mass is greater than the mass of a body, the calculated motion is divergent because of its self-similar structure. This study showed that the above problem is an inherent problem. This problem of self-similar structure may arise in the equation of motion in which the fluid dynamic forces are treated as external forces on the right hand side of the equation. A reconfiguration technique for the equation of motion using pseudo-added-mass was proposed to resolve the self-similar structure problem; specifically for the case when the fluid force is expressed by integration of the fluid pressure.*

KEY WORDS: Self-similarity; Added mass; Equation of motion; CFD; Numerical stability; Renormalization.

INTRODUCTION

If we want to analyze the motion of a body in a fluid, we should analyze the rigid-body dynamics of the body and the fluid dynamics of the surrounding fluid. Traditionally, the potential theory has been used for the analysis of the flow of fluid in studying the motion of ships. Added mass, which is the mass of surrounding fluid moving together with a ship, was treated as part of the virtual mass of that ship, and the wave damping forces were treated as the damping of the body. The main external force comes from incident waves, so gravity wave dynamics had to be solved, and the resultant pressure was integrated over the body surface to form the wave external force.

Currently, in analyzing ship motion, besides the potential theory, Computational Fluid Dynamics (CFD) based on the Navier-Stokes equation is being used; owing to the development of computers. One of the main differences between the potential theory and CFD is the treatment of added mass. In the potential theory, the added mass can be expressed explicitly, so one obtains the added mass, adds this to the mass of the ship, and then analyzes the ship motion. However using CFD, the ship motion analysis is done by first solving the flow of surrounding fluid for a given ship motion; then adding the pressure forces acting on the ship surface into the external forces of the equation of motion. This is because the inertia forces due to added mass cannot be expressed explicitly, so one cannot help but treat it as external force. Thus the difference is whether the added mass can be treated as ship-mass or be included in external forces. Using CFD, an assumption of convergence is made; that is, there is no problem if the time interval for time integration is reduced sufficiently, even if the inertia force due to added mass is treated as external force.

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In this study, we revisited the property of added mass, investigated its property in the equation of motion, and the problems that can take place. Our work revealed that the equation of motion has a self-similar structure if the forces due to added mass are treated as external forces; then the solution can diverge. In differential equation, the highest order differential term has the most important role in the equation. We say that the equation has a self-similarity, if the variable to be sought(the highest order differential term) is expressed by forcing terms including that variable itself. In a numerical example, it was shown that the solution can be divergent even if the time interval is reduced sufficiently. This divergent solution may be a false solution due to the self-similarity of the equation, not due to the dynamic property. When using the non-linear potential theory or CFD based on the Navier-Stokes equation for analysis of the surrounding fluid, the reconfiguration technique using 'pseudo-added-mass' to resolve the problem of self-similar structure was proposed. The pseudo-added-mass is indeed mass that is pseudo-added in order to ensure a convergence of the solution.

ADDED MASS REVISITED

The concept of added mass or virtual mass has appeared in the literature since 1828, when Friedrich Bessel described the motion of a pendulum in fluid domain. The period was longer than that in the air, and this phenomenon was explained as being due to an increase in the effective mass by the surrounding fluid. Later, the concept of added mass played an important role in the analysis of the motion of a body, and the added mass was obtained analytically for simple shapes. Surely, the concept of added mass has been used in ship motion analysis, but the added mass could only be obtained after the shape of the ship was modified to a less accurate, simpler shape.

The added mass of more ship-like shapes was obtained by Lewis (1929). He used the conformal mapping technique to mathematically represent the shape of the ship cross section; a result that is now called the Lewis form. He used added mass in his analysis of the vibration of a ship. Hess and Smith (1962) proposed the numerical method to calculate the potential flow around a 3-dimensional body of arbitrary shape. The added mass of a body floating on a free surface has different characteristics from those of a body in unbounded fluid. The formulations on this problem were made by Ursell (1949), and John (1949; 1950). Ursell (1949) analyzed the motion of a circular cylinder on a free surface by obtaining the added mass using the multi-pole expansion method. Later, Tasai (1959) developed the multi-pole expansion method further to obtain the added mass of a Lewis form on a free surface. Now, the added mass of ship-like section could be calculated. Frank (1967) proposed a calculation method for added mass of an arbitrary shape, and more developments on this method made it applicable to 3-dimensional shapes. Newman (1978) gave a good account of the history and efforts toward representing the motion of a ship.

In order to describe the motion of a body in the fluid domain, the two systems of dynamics are needed: rigid-body and fluid dynamics. For convenience, the explanation will be given for a 1-dimensional problem, but this logic can be applied to 2 and 3-dimensional problems as well. According to rigid-body dynamics, Newton's law appears in the form:

$$\frac{d}{dt}(mv) = f(v, x, t) \tag{1}$$

where m is the mass of a body, and v, x are the velocity and position of the body, respectively. If we assume that the mass of the body will not alter in time, the equation becomes:

$$m\dot{v} = f(v, x, t) \tag{2}$$

If we suppose that the body is in the fluid domain, the force in the above equation includes the forces due to the pressure of fluid and other forces f'; such that:

$$m\dot{v} = f'(v, x, t) + \int_{S_B} p n_x dS \tag{3}$$