©SNAK, 2015

Int. J. Nav. Archit. Ocean Eng. (2015) 7:324~345
http://dx.doi.org/10.1515/ijnaoe-2015-0023
pISSN: 2092-6782, eISSN: 2092-6790

A new finite element formulation for vibration analysis of thick plates

Ivo Senjanović[1], Nikola Vladimir[1] and Dae Seung Cho[2]

[1]*University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia*
[2]*Pusan National University, Dept. of Naval Architecture and Ocean Engineering, Busan, Korea*

ABSTRACT: *A new procedure for determining properties of thick plate finite elements, based on the modified Mindlin theory for moderately thick plate, is presented. Bending deflection is used as a potential function for the definition of total (bending and shear) deflection and angles of cross-section rotations. As a result of the introduced interdependence among displacements, the shear locking problem, present and solved in known finite element formulations, is avoided. Natural vibration analysis of rectangular plate, utilizing the proposed four-node quadrilateral finite element, shows higher accuracy than the sophisticated finite elements incorporated in some commercial software. In addition, the relation between thick and thin finite element properties is established, and compared with those in relevant literature.*

KEY WORDS**:** Mindlin plate theory; Finite element formulation; Thick-thin plate relation; Vibration analysis; Shear locking.

INTRODUCTION

In plate theory, two mathematical models are distinguished, the well-known Kirchhoff thin plate and the Mindlin thick plate theory. In the former, shear influence on deflection is small and is therefore ignored. This theory is very well developed and the achievements are presented in Szilard's fundamental book (Szilard, 2004). The dynamic behaviour of thick plate is quite a complex problem, due to shear influence and rotary inertia, and is still an interesting subject of investigation. The first works are those of Reissner and Mindlin (Reissner, 1945; Mindlin, 1951), in which it is assumed that the plate cross-section remains a plane but not normal to the plate middle surface. As a result, the Mindlin theory deals with a system of three differential equations of motion with plate deflection and two angles of rotation of cross-section as unknown variables. This system is the starting point for all later developed variants. In the meantime, a large number of articles have been published and a comprehensive survey of literature up to 1994 can be found in (Liew et al., 1995).

Generally, there are two main approaches for solving the problem of thick plate natural vibrations, i.e. analytical methods for the solution of differential equations of motion, and numerical procedures based on the Rayleigh-Ritz energy method as well as the Finite Element Method (FEM). Different analytical methods are known depending on which functions are kept as fundamental ones in the reduction of the system of differential equations of motion. Some methods operate with three, two or even one function, for instance Wang (1994), Shimpi and Patel (2006) Endo and Kimura (2007) and Xing and Liu (2009), respectively. The application of analytical methods is relatively simple for simply supported plates and plates with simply supported two opposite edges. A sophisticated closed-form solution is derived in Xing and Liu (2009) for plate vibration analysis with any

Corresponding author: *Nikola Vladimir*, e-mail: *nikola.vladimir@fsb.hr*

combination of simply supported and clamped edges.

The Rayleigh-Ritz method is widely used for arbitrary boundary conditions as well as for elastically supported edges. Accuracy depends on the chosen set of orthogonal functions for the assumed natural modes. For that purpose two dimensional polynomials can be used Liew et al. (1993), or functions of static Timoshenko beam deflection Dawe and Roufaeil (1980), Cheung and Zhou (2000). Efficient solution is also achieved applying the assumed mode method with static Timoshenko beam functions to thick bare plate Kim et al. (2012) as well as for some more complex problems (Cho et al., 2013; 2014).

The finite element method is a very powerful tool for the analysis of any problem (linear, nonlinear, static and dynamic) of engineering structures with a complicated configuration. Several finite elements for Mindlin plate have been developed and incorporated in commercial FEM software. They deal with three displacement fields, i.e. deflection and two rotations. Due to the impossibility to prescribe correct interdependence among deflection and rotations, the same order polynomials for the interpolation of all displacements are used. Consequently, so-called shear locking phenomenon arises since in transition from thick to thin plate, it is not possible to capture pure bending modes and zero shear strain constraints. There are a few procedures for solving shear locking problem in the FEM analysis, which are referred to in Falsone and Settineri (2012): reduced integration for shear terms (Zienkiewicz et al., 1971; Hughes et al., 1977), which is commonly used in commercial software; mixed formulation of hybrid finite elements (Lee and Wong, 1982; Auricchio and Taylor, 1995; Lovadina, 1998); Assumed Natural Strain (Hughes and Tezduyar, 1981; Bathe, 1996; Zienkiewicz and Taylor, 2000); and Discrete Shear Gap (DSG) (Bletzinger et al., 2000), and its combination with the mesh-free procedure (Liu et al., 2009; Nguyen-Xuan et al., 2010). Recently, a new shear locking free finite element formulation for static analysis of thick plate has been proposed in (Falsone and Settineri, 2012) based on an extension of the well-known Kirchhoff thin plate theory. The so-called fictitious deflection is used as a basic function, by which the other kinematic and static quantities are determined. It is also necessary to mention the worthwhile formulation of the mixed FEM and the Differential Quadrature Method (DQM) for longitudinal and transverse plate direction, respectively (Eftekhari and Jafari, 2013).

Motivated by the state of the art described above, in the present paper a new finite element formulation is proposed for thick plate vibration analysis. It is based on a new moderately thick plate theory presented in (Senjanović et al., 2013a; 2013b), where a system of governing differential equations of motion is reduced to one equation with bending deflection as an unknown function. It is actually a potential function by which total deflection and angles of rotations are determined, as well as bending and shear strains and sectional and inertia forces. This plate theory actually represents an extension of the modified Timoshenko beam theory (Senjanović and Fan, 1989; Senjanović and Grubišić, 1991; Senjanović et al., 2009; Senjanović and Vladimir, 2013). Due to strong interdependence among deflection and rotations, the shear locking phenomenon does not occur. Finite element stiffness and mass matrices are determined by employing an ordinary variational formulation (Zienkiewicz and Taylor, 2000).

OUTLINE OF NEW PLATE THEORY

Deformation of a thick rectangular plate is considered in the Cartesian coordinate system in Fig. 1. By following the idea from the modified Timoshenko beam theory, total deflection is decomposed into bending deflection and shear deflection (Senjanović et al., 2013a; 2013b)

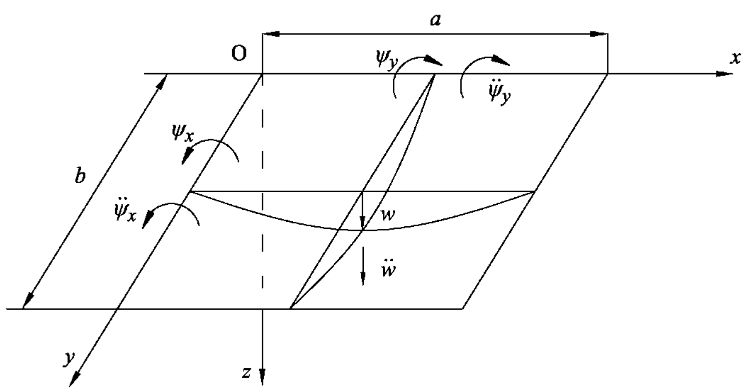

Fig. 1 Displacements of rectangular plate.