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A calculation method for finite depth free-surface green function

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ABSTRACT: *An improved boundary element method is presented for numerical analysis of hydrodynamic behavior of marine structures. A new algorithm for numerical solution of the finite depth free-surface Green function in three dimensions is developed based on multiple series representations. The whole range of the key parameter R/h is divided into four regions, within which different representation is used to achieve fast convergence. The well-known epsilon algorithm is also adopted to accelerate the convergence. The critical convergence criteria for each representation are investigated and provided. The proposed method is validated by several well-documented benchmark problems.*

***KEY WORDS*:** Boundary element method; Free-surface green function; Fast numerical solution.

INTRODUCTION

Prediction of the hydrodynamic performance of a floating platform for ocean renewable energy has been an important issue in recent years. Although extensive researches have been carried out on offshore oil platforms, many new research topics arise in the development of offshore renewable energy platform due to the different cost requirement and the existence of wind turbines on the deck. The present research is aiming to develop a new and efficient analysis tool for such ocean renewable energy platforms. A Boundary Element Method (BEM) has been developed for calculation of hydrodynamic loads on the platform. This paper presents a new algorithm for numerical solution of the finite depth free-surface Green function in three dimensions which is used in the proposed method.

Among various numerical methods nowadays, the boundary element method based on potential flow theory is one of the most popular methods due to its fast and efficient computation. Under the assumption that the fluid is incompressible, inviscid and irrotational, the BEM uses an appropriate Green function together with Green's theorem to formulate the boundary integral equations with appropriate boundary conditions. Numerical algorithms of BEM include discretization of the body surface into some low-order elements or higher-order elements, distribution of the sources/dipoles on the submerged part of the hull surface, and calculation of the influence coefficients between arbitrary two sources/dipoles. In this method, the numerical solution of the free-surface Green function consumes a large portion of CPU time.

An alternative approach for simple-shape bodies is the multipole expansion method which was first investigated by Ursell (1949) and Havelock (1955). These multipoles, generally contain two parts, i.e., the wave-free term which vanishes in the far-field away from the structure, and the wave-source term which travels radially outwards at infinity (Liu et al., 2012). While the

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wave-free term consists of some infinite series which are easy for evaluation, the wave-source term including the free-surface Green function is hard to be numerically calculated. Therefore, no matter which method is used, efficient numerical implementation of the free-surface Green function remains the major task (Newman, 1992).

Numerous work has been done on the subject of the free-surface Green function. John (1949; 1950) first derived a variety of representations in both two and three dimensions with the consideration of both infinite and finite water depth. Infinite series expansion method for Green function under finite depth was presented by Wehausen and Laitone (1960). Thorne (1953), Mei (1983) and Linton and McIver (2001) also have important contribution. As to its numerical evaluation, several scholars have done many important works from 1980s, in which Newman (Newman, 1985; 1992) developed several efficient algorithms for both frequency domain and time domain Green functions in zero forward speed, which have been wildly used in this field. Pidcock (1985) derived several valuable expressions of the frequency domain Green function in finite depth, using series expansions. Linton (1999) suggested a novel set of representations for Green functions of Laplace's equation, which converges rapidly provided the optimal value of a parameter *a* is properly given.

There are mainly three schemes for solution of Green functions: separation of the local and the far-field component and tabulation; numerical integration and Chebyshev approximation; utilization of series representations and acceleration. The first scheme usually needs to seek elaborate mathematical derivations. The second scheme requires computation of the principal integral, which is usually not numerically stable by an adaptive integration method. In addition, it is difficult to choose the optimal subdivision of regions for Chebyshev approximation. The last scheme is much easier for programming, but requires different series representations in different regions with respect to the ratio R/h (R is the horizontal distance between source and field point, and h is the water depth). As to the authors' knowledge, there are still rooms to do research on improvement of methods based on the first or the third scheme for evaluating finite-depth free surface Green function. John's eigenfunction expansion representation (John, 1950) converges rapidly except for relatively small R/h. Pidcock's representations (Pidcock, 1985) are invalid for large wave numbers when R/h is small, and it is difficult to find an appropriate convergence criterion for the summation of the series terms. Linton's rapidly convergent representation (Linton, 1999) converges in the whole domain of R/h. However, it needs careful selection of the parameter a according to the ratio R/h and the wave number v, which is not a trivial work.

In this paper, we propose an efficient boundary element method that can be used to predict hydrodynamic property of an ocean renewable energy platform. The major contribution of this research is that a new algorithm for numerical solution method of the free surface Green function is developed. According to the ratio R/h, the computation domain is divided into four subdomains in which different series expansion scheme is used. The well-known epsilon algorithm is adopted to accelerate the convergence. A rapidly convergent expression scheme, which is based on the Chebyshev approximation method, is derived by substitution of its counterpart into Pidcock's representation. The optimal selection method of the parameter a is also given for small R/h in Linton's representation. In addition, the critical convergence criteria for each representation are also studied and some practical linear approximations are given. To remove the irregular frequencies in the wave interaction problem of surface-piercing bodies, the partial extended boundary integral equation method originates from Lau and Hearn (1989) is applied. Several validation examples are presented to demonstrate the performance of the method. Finally, as an example of engineering application, numerical results by the proposed method are shown with the comparison to other numerical methods.

SOLUTION OF THE BOUNDARY VALUE PROBLEMS

Mixed-source/dipole BEM formulation

Denote the coordinate system to be right-handed Cartesian coordinate system (x,y,z) with its x-y plane taken as the undisturbed sea level and the z-axis taken vertically upwards. Consider an incident wave transmitting along the direction positive, and a rigid body floating or being submerged in water of constant depth with a free surface. Under the usual assumptions of an inviscid, irrorational and incompressible flow, the problem can be described by a velocity potential

$$\Phi(x,y,z,t) = \mathrm{Re}\left(\phi_0(x,y,z)e^{-\mathrm{i}\omega t}\right) + \mathrm{Re}\left(-\mathrm{i}\omega\sum_{j=1}^{6}\phi_j(x,y,z)\xi_j e^{-\mathrm{i}\omega t}\right) + \mathrm{Re}\left(\phi_7(x,y,z)e^{-\mathrm{i}\omega t}\right), \tag{1}$$