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Moment of inertia of liquid in a tank

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ABSTRACT: *In this study, the inertial properties of fully filled liquid in a tank were studied based on the potential theory. The analytic solution was obtained for the rectangular tank, and the numerical solutions using Green's 2nd identity were obtained for other shapes. The inertia of liquid behaves like solid in recti-linear acceleration. But under rotational acceleration, the moment of inertia of liquid becomes small compared to that of solid. The shapes of tank investigated in this study were ellipse, rectangle, hexagon, and octagon with various aspect ratios. The numerical solutions were compared with analytic solution, and an ad hoc semi-analytical approximate formula is proposed herein and this formula gives very good predictions for the moment of inertia of the liquid in a tank of several different geometrical shapes. The results of this study will be useful in analyzing of the motion of LNG/LPG tanker, liquid cargo ship, and damaged ship.*

KEY WORDS: Inertia of liquid; Sloshing; Roll moment of inertia; Liquid in a tank; Fully filled tank.

INTRODUCTION

If a ship has floodwater and/or liquid cargo inside, the liquid influences the ship motions. The mechanism of this influence depends on the existence of free surface in the tank containing the liquid. If the free surface exists, the motion of liquid is relatively free compared with when no free surface exists. In statics with free surface, the center of gravity can move and change the restoring force, which is called the free surface effect. However, when a ship motion problem is considered in a typical ocean environment, the motion of the ship induces the motion of the liquid in a tank, and at the same time the motion of the liquid in the tank also imparts the hydrodynamic excitations to the motion of the ship. This is fully coupled problem. When we further assume the tank to be completely filled, i.e., no free surface in the tank, the effect of the liquid in the tank can be treated as a solid for the recti-linear motions of the ship: this particular case reduces to be trivial since there is no liquid motion in the tank with respect to the ship motions. However, when a ship undergoes angular motions, then the liquid in the tank can no longer to be treated as a solid for the computation of the moment of inertia of that.

It is well known that the study of the liquid in a ship was started by W. Froude in 1874 as he conducted the model test to see the effectiveness of free surface tank as a ship stabilizer. In 1910, Frahm introduced U-tube as a stabilizing device and it was widely used (Bhattacharyya, 1978). But in 1950's the free surface tank was revived and used in many naval vessels, because it has the added advantage varying the natural frequency of the tank by changing the water level and thus accommodating the changes in ship's metacentric height. During that period and after, the characteristics of free surface tank were investigated by many researchers; Verhagen and van Wijngaarden (1965) studied the non-linear hydrodynamics when the average height of

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free surface was low so the free surface elevation to water depth ratio was high, and Van Den Bosch et al. (1965) gave the results of their studies on the performance of free surface tank as a stabilizer.

With the progress of aeronautics, many studies on the movements of fuel have been done. Especially in 1950's and 1960's, a lot of researches have been done on the control of missiles and rockets to take into account the effect of the fuel, of which the amount was diminishing and the mass and inertia had been changing continuously (Graham and Rodriguez, 1952; Abramson, 1966; Roberts et al., 1966). The main focus was on finding the resonance mode of fuel tank that has free surface, and finding the equivalent mass-spring-damper system that has the same hydrodynamic properties. The linear analysis was sufficient in the dynamics for the control of missiles and rockets (Ibrahim, 2005).

Currently, the researches on the liquid cargo are mainly focused on the analysis of the sloshing phenomena. Sloshing phenomenon did not cause problems in crude oil takers, but caused problems in purified oil tankers and LNG(liquefied natural gas) tankers. It has effects on the ship motion, and also damages on the top of tank because of the excessive pressure made by the hydrodynamics of free surface in a restricted area (Kim et al., 2013; Ahn et al., 2013). Other researches include sloshing under micro-gravity in space to be used in the field of the analysis of the space vehicles and the control of them (Helder, 2005) and sloshing in seismic conditions as the earthquake often demolish the liquid tank on land (Housner, 1954; Dogangun and Livaoglu, 2008).

When the free surface exists, as mentioned previously, the effects of liquid cargo have been taken into account by using the method of coupling the ship motion dynamics and liquid hydrodynamics, which are solved by one of the proven methods such as the equivalent mass-spring-damper dynamics, potential flow hydrodynamics and computational fluid dynamics (CFD). In the case of fully filled liquid, the liquid is treated as solid and is included in the ship's mass in many studies for the motion dynamics of ships. However, the inertia of liquid has difference compared with that of solid, especially in rotational acceleration. The inertia of the liquid in a fully filled tank was studied in this paper. The formulations on the liquid dynamics and inertia properties were solved, and the analysis of inertia properties has been done. As a result, for the rotational acceleration, the moment of inertia of liquid turns out to be small compared to that of solid, while the liquid acts like solid in recti-linear acceleration. The numerical solution was compared with the analytic solution, and the formula that accurately estimates the moment of inertia has been proposed.

INERTIA OF LIQUID

The inertia of material in a ship must be treated in different ways according to whether it is moving with the ship or not. In the case of moving with the ship, the ship's dynamic can be analyzed under the condition that the mass of that material is included in ship's mass. But in the case that the motion of material is different with that of the ship, the dynamics of the material and the ship must be solved concurrently and they must be coupled together to interact with each other. For example as in Fig.1 the material is on the deck of a ship and not fixed, if the downward acceleration is less than the gravitational acceleration it moves with the ship, but if the downward acceleration is greater than gravity the material fly and the mass of material is not included in the ship's mass. For another example as in Fig. 2, when the circular cylinder accelerates in rotation, the inner liquid does not accelerate if the viscosity is neglected. In this case, the moment of inertia comes only from the circular tank and not from the liquid in it.

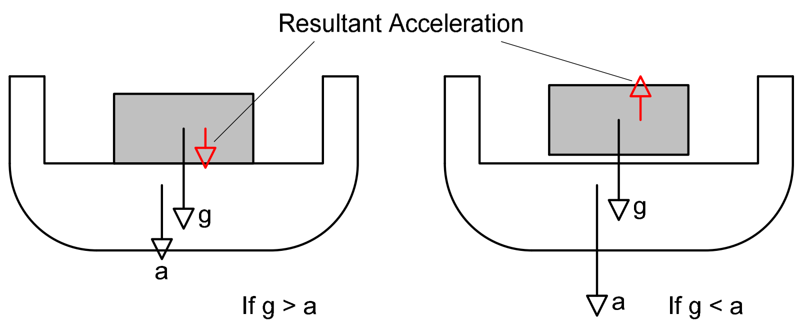

Fig. 1 Resultant acceleration of rigid material on deck.