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Prediction of propagated wave profiles based on point measurement

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ABSTRACT: *This study presents the prediction of propagated wave profiles using the wave information at a fixed point. The fixed points can be fixed in either space or time. Wave information based on the linear wave theory can be expressed by Fredholm integral equation of the first kinds. The discretized matrix equation is usually an ill-conditioned system. Tikhonov regularization was applied to the ill-conditioned system to overcome instability of the system. The regularization parameter is calculated by using the L-curve method. The numerical results are compared with the experimental results. The analysis of the numerical computation shows that the Tikhonov regularization method is useful.*

KEY WORDS: Wave profile; Singular value decomposition; Ill-conditioned matrix; Tikhonov regularization; Regularization parameter; L-curve; Gaussian wave packet.

INTRODUCTION

Sea loads considerably influence on offshore structures' safety, safety for workers, and efficiency of work in the ocean. Sea loads are the major influencing factors that have to be carefully considered when the structures are in operation. Many studies on sea loads are in progress. Especially, wave induced loads are the dominant factors to be considered. In order to estimate the wave induced loads, the accurate prediction of wave itself is critical.

In order to predict accurate wave field around offshore structure, the mechanism of wave propagation should be established. Wave propagation prediction has many practical applications. Just name a few of them, prediction of wave near floating structures are necessary for the safe operation of the structures. We can get benefited from estimating the safe time period for landing of helicopters on offshore structures or on warships. The damage caused by sloshing can be avoided if one can estimate the sloshing load from wave prediction by taking countermeasures.

At present, the usual practice of wave measurement is using wave gauge at a fixed point. Using the linear wave theory and the principle of superposition, the integral equation can be derived from the time series data at the fixed point. The generated integral equation is called Fredholm integral equation of the first kind. With integral equation discretized by wave number and time, the equation which relates wave amplitude and wave spectrum can be formulated. Wave amplitude spectrum can be obtained by solving the constructed matrix and using wave amplitude spectrum. Wave elevation at arbitrary points and time can be estimated. The discretized integral equation is mostly ill-conditioned which leads to unstable solutions. In order to estimate accurate wave elevation at target point, Tikhonov regularization method is introduced (Fridman, 1956; Isakov, 1998; Kammerer and Nashed, 1972; Tikhonov, 1963). When it comes to the Tikhonov regularization method, choosing optimized regularization parameter is essential to ensure the stability and accuracy of matrix. Present study employed the L-curve method.

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In this study, Gaussian wave packet is introduced to realize the incoming waves. Numerical simulation of the Gaussian wave has been conducted. The accuracy was evaluated by performing numerical and analytic works on this. Furthermore, the wave propagation experiment in wave tank is conducted. The wave time series has been measured at a fixed point. By analyzing the data, the wave field at the target point is estimated. Again, the ill-posed system is solved by applying Tikhonnov regularization method. The regularization parameter was estimated by the L-curve criterion which was successfully introduced by Lawson and Hansen (1974). The estimated wave time series was compared with the measured time series. The present scheme can be extended to multidirectional short crested sea if a spreading function is introduced. The present study is limited to linear dispersive waves due to its mathematical formulation. Therefore one should come up with non-dispersive scheme to deal with nonlinear wave field.

WAVE PROPAGATION MODEL

Assuming an ideal fluid, with its motion irrotational for a small amplitude wave, the wave elevation can be written as

$$\eta(x,t) = \mathrm{Re}\left[a e^{i(kx-\omega t)} \right] \tag{1}$$

where a is the amplitude of the wave, k is the wave number, and ω is the wave frequency. The linear superposition of the elementary solutions gives us an integral form as follows

$$\eta(x,t) = \mathrm{Re}\left[\int_{-\infty}^{\infty} a(k) e^{i(kx-\omega t)} dk \right] \tag{2}$$

where $a(k)$ is wave amplitude spectrum. The wave elevations in different positions and time can be obtained. If the amplitude spectrum, $a(k)$ is given in Eq. (2). However, it is not always straightforward to obtain the analytic closed form solution to the integral defined in Eq. (2) although the function $a(k)$ is specified. When a finite depth of water is considered, it is not possible to obtain a closed form analytic solution to the integral due to the dispersion relation, i.e., the characteristic relation between the wave number and the frequency of the waves. The well-known linear dispersion relation is

$$\omega = \sqrt{gk \tanh kh} \tag{3}$$

where h is the depth of water. Since the closed form solution is not available for this case, we can seek an approximate solution.

A fredholm integral equation of the first kind; ill-posed problem

Eq. (2) is a Fredholm integral equation of the first kind. The wave elevation shown in the left hand side of the Eq. (2) will be a given function from measurement. The term $a(k)$ is an unknown function we are after. It is well known that the Fredholm integral equations of the first kind are ill-posed. It is in order to explain the concept of well-posed problem first before we get to the ill-posed problem. The problem given is well-posed if

(1) The solution exists
(2) The solution is unique
(3) The small change in the right hand side of Eq. (2) causes small change in the solution.

If the given problem is not well-posed, it is said to be ill-posed. Therefore the problem we have formulated is an ill-posed problem. The remedy for the ill-posed problem is regularization which will be covered in a latter chapter. If we discretize the Eq. (2) the matrix we encounter becomes ill-conditioned matrix.

Discretization of the integral equation

A typical and most often used method of approximating an integral is first to discretize the integrand given in a continuous function into a finite number of segments and then the original integral is reduced to a sum of the integrals of the segments. Then the wave elevation in Eq. (2) can be approximated by