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Simplified formulas of heave added mass coefficients at high frequency for various two-dimensional bodies in a finite water depth

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ABSTRACT: *The aim of this study is to develop a simplified formula for added mass coefficients of a two-dimensional floating body moving vertically in a finite water depth. Floating bodies with various sectional areas may represent simplified structure sections transformed by Lewis form, and can be used for floating body motion analysis using strip theory or another relevant method. Since the added mass of a floating body varies with wave frequency and water depth, a correction factor is developed to take these effects into account. Using a developed two-dimensional numerical wave tank technique, the reference added masses are calculated for various water depths at high frequency, and used them as basis values to formulate the correction factors. To verify the effectiveness of the developed formulas, the predicted heave added mass coefficients for various wetted body sections and wave frequencies are compared with numerical results from the Numerical Wave Tank (NWT) technique.*

KEY WORDS: Added mass coefficient; Simplified formula; Finite water depth; Sea bottom effect; Simple geometric body.

INTRODUCTION

The accurate prediction of hydrodynamic coefficients, such as added mass and radiation damping coefficients, is crucial in analyzing the motion response of a floating structure in water waves. Since ships and various ocean structures are often operated in a finite water depth, the precise calculation of hydrodynamic coefficients corresponding to various wave frequencies and water depths is getting important. In particular, the heave added mass of a large submerged or floating body plays an important role in determining body responses in the shallow water depth. When the wave frequency increases, the added mass coefficients gradually increases and converges to a constant value, while the radiation wave damping decreases then goes to zero. Therefore, the relative effect of added mass is much greater than that of damping coefficient. The change of heave added mass coefficient for various wave frequencies and water depths has to be considered in analyzing ship and ocean structure maneuverability and operation in finite water depth regions such as harbor and canal.

Since the 1940s, various calculation methods for the hydrodynamic coefficients of a floating structure have been developed. In the early days, Ursell (1948) first calculated the added mass and wave damping coefficient of a cylinder in an infinite water depth using the velocity potential and stream functions. Later, Frank (1968) also calculated the hydrodynamic

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coefficients in the same conditions by solving the boundary value problem in which source singularities are distributed over the submerged portion of the body. After their pioneering work, numerous studies on the calculation of hydrodynamic coefficients with different conditions have been conducted using various numerical methods (Shen et al., 2005; Zhou et al., 2005; Sutulo et al., 2010).

When the hydrodynamic coefficients of an arbitrary body shape are to be computed in a finite water depth, the use of complicated mathematical analysis or state-of-the-art numerical tools are required, which can be expensive and time-consuming. Hence, an empirical formula for added mass has been developed and used for ship motion analysis because of its easy calculation and convenient application. Since the empirical formula, used in practical shipbuilding and ocean engineering design, was developed for conditions of infinite water depth and infinite wave frequency, the formula cannot reflect changes of added mass due to proximity to the sea bottom or the effect of free surfaces. Thus, the existing empirical formula for added mass should only be applied under certain conditions.

A simple formula for added mass prediction for various body sections that considers the effects of the sea bottom and wave frequencies is required for a quick analysis of a floating body response in shallow water, especially in the early design stage.

In this study, based on the basic study of Koo and Kim (2013) for semicircular body, we developed a simplified formula of heave added mass coefficients for various two-dimensional body sections in a finite water depth. This formula can reflect changes in added mass due to the effects of the sea bottom and wave frequencies. The added mass predicted by the simplified formula was compared with the results calculated by NWT, and the validity of the formula was confirmed for various wave frequencies and water depths.

MATHEMATICAL FORMULATION OF NWT TECHNIQUE

To calculate the basis values of the added masses corresponding to various water depths at high frequency, a two-dimensional NWT developed by numerous researchers (see the review paper on recent research and development of the NWT technique by Kim et al., 1999) was utilized to solve the boundary value problem in a computational domain filled with an inviscid, incompressible, and irrotational fluid. The NWT used in this study was based on boundary element method with constant Rankine panels and linear wave theory. In the paper, we briefly introduce the mathematical formulation of the NWT technique for the convenience of readers.

As a governing equation in the computational fluid domain, the Laplace equation ($\nabla^2\phi = 0$) can be formulated using the velocity potential ($\Phi(x,z;t) = \mathrm{Re}[\phi(x,z)e^{-i\omega t}]$, ω =wave frequency). Using the Green function (G_{ij}), the Laplace equation can be transformed into the boundary integral equation (Eq. (1)), to which the corresponding boundary conditions over the entire computational domain can be applied.

$$\alpha\phi_i = \iint_{\Omega} \left(G_{ij}\frac{\partial\phi_j}{\partial n} - \phi_j\frac{\partial G_{ij}}{\partial n}\right)ds \tag{1}$$

where the solid angle α was equal to 0.5 on the smoothed surface in this study, n is the unit normal vector, $G_{ij}(x_i, y_i, x_j, y_j) = -(1/2\pi)\ln R$ for a two-dimensional problem, and R is the distance between the source $(x_j,\ y_j)$ and field points $(x_i,\ y_i)$ placed on the computational boundaries. To solve the boundary integral equation, the linearized free surface boundary condition ($\partial\phi/\partial z - \omega^2\phi/g = 0$ on z=0, g is gravitational acceleration) and rigid boundary condition ($\partial\phi/\partial n = 0$ on $z = -h$) are defined. As an open boundary condition at both vertical-end boundaries of the domain, the radiation boundary condition is described as in the frequency domain:

$$\frac{\partial\phi}{\partial n} = \frac{\partial\phi}{\partial x}n_x = ik\phi(\vec{x}) \tag{2}$$

where n_x is the horizontal component of the unit normal vector n, and k is the wave number. The body boundary condition for the wave radiation problem is described as: