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Energy flow analysis of out-of-plane vibration in coplanar coupled finite Mindlin plates

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ABSTRACT: *In this paper, an Energy Flow Analysis (EFA) for coplanar coupled Mindlin plates was performed to estimate their dynamic responses at high frequencies. Mindlin plate theory can consider the effects of shear distortion and rotatory inertia, which are very important at high frequencies. For EFA for coplanar coupled Mindlin plates, the wave transmission and reflection relationship for progressing out-of-plane waves (out-of-plane shear wave, bending dominant flexural wave, and shear dominant flexural wave) in coplanar coupled Mindlin plates was newly derived. To verify the validity of the EFA results, numerical analyses were performed for various cases where coplanar coupled Mindlin plates are excited by a harmonic point force, and the energy flow solutions for coplanar coupled Mindlin plates were compared with the classical solutions in the various conditions.*

KEY WORDS: Energy flow analysis (EFA); Mindlin plate theory (MPT); Kirchhoff plate theory (KPT); Out-of-plane propagating wave; Wave transmission analysis (WTA).

INTRODUCTION

The remarkable development of offshore oil-and-gas, automotive, electronics, and aerospace industries has facilitated an ever-growing interest in high-frequency noise and vibration. Traditional Finite Element Analysis (FEA) and Boundary Element Analysis (BEA) are known to be unsuitable for predicting structure-borne noise of built-up structures at high frequencies for several reasons. First, because of the relatively very small size of the wavelengths with respect to the size of each structural component at high frequencies, FEA and BEA based on traditional displacement solutions are potentially computationally expensive and time consuming to develop and to check out analytical models for complex structures (Cho et al., 2010; Cho et al., 2014). Second, deterministic approaches such as FEA and BEA are not practical for the prediction of vibrational responses in built-up structures at high frequencies. In particular, the vibrational behavior of a structure at high frequencies becomes increasingly dependent on fine structural details such as structural joints and boundaries, which cannot be mathematically represented with sufficient accuracy in these frequencies ranges (Fahy, 1990; Cremer and Heckl, 1998). Also, in even nominally identical structures, the fabrication tolerances allowed during manufacturing processes in mechanical industries cause differences in high-frequency vibrational and acoustic responses (Wood and Joachim, 1987; Kompella and Bernhard, 1993).

The statistical approach has been widely used in various industries as an alternative to deterministic approaches. Statistical Energy Analysis (SEA), the representative analytic method of statistical approaches, can effectively predict the space- and frequency-averaged behavior of built-up structures at high frequencies where the modal overlap of structural components is

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high (Lyon and Dejong, 1995; Lyon and Eichler, 1964; Lyon, 1995). However, because SEA uses lumped dynamic characteristics and has a linear algebraic type power balance equation, SEA can predict only a single acoustic or vibrational response value for each subsystem of a built-up structure. Therefore, SEA cannot consider various local treatments within each structural component and cannot directly use the finite element model or boundary element model which is inevitably produced during the general Noise, Vibration and Harshness (NVH) analytic process. Additionally, the hybrid FE-SEA has been studied to improve the reliability of analytic results at mid frequencies (Langley and Bremner, 1999; Langley and Cordioli, 2009).

Energy Flow Analysis (EFA), elsewhere termed Power Flow Analysis (PFA), is a representative statistical method that has been suggested to overcome these weaknesses of the general SEA. The heat conduction type governing differential equation of EFA and the wave transmission analysis of infinite structures make it possible to predict locally space- and frequency-averaged vibrational behavior of arbitrary built-up structures. EFA was first introduced by Belov et al. (1977). EFA investigations can be classified into three main research fields. The first research field focuses on the derivation of an energy governing equation of analytic dynamic systems. Many earlier researchers expanded the applicable region of EFA to include elementary dynamic systems such as rod, Euler-Bernoulli beam, membrane, and Kirchhoff plate (Nefske and Sung, 1989; Wohlever, 1989; Wohlever and Bernhard, 1992; Bouthier, 1992; Bouthier and Bernhard, 1992; Bouthier and Bernhard, 1995; Bouthier et al. 1999). Park et al. (2001) developed an energy flow model for in-plane waves of a finite plate, and recently, derived heat conduction type governing equations for out-of-plane waves in a Timoshenko beam and Mindlin plate, which can consider the rotatory inertia and shear deformation effects at high frequencies (Park and Hong, 2006a; 2006b; 2008). The second research field involves the Wave Transmission Analysis (WTA) among the propagating waves that exist in various structural systems for the energy flow prediction of coupled vibrational and acoustic systems. Park et al. derived the wave transfer relationship among the flexural and in-plane waves existing in the Kirchhoff plate (Park et al., 2001). Seo et al. then developed the wave transfer model for reinforced beam-plate coupled structures (Seo et al., 2002), and derived wave transmission and reflection coefficients among propagating flexural waves with a reinforced beam in the junction of a coplanar coupled Kirchhoff plate. Kang et al. performed the WTA and EFA of penetration beam-plate coupled structures (Kang, 2001; Song et al., 2011), and Park et al. studied the WTA and EFA of coupled Timoshenko structures (Park and Hong, 2006a; 2006b). Recently, Park derived the simple power transfer relationship of two coplanar coupled semi-infinite Mindlin plate (Park, 2013). The third research field involves the application of EFA to numerical techniques such as the Finite Element Method (FEM) and Boundary Element Method (BEM) for the EFA of arbitrary multi-dimensional built-up structures. Nefske and Sung suggested the Energy Flow Finite Element Analysis (EFFEA) of coupled beam structures (Nefske and Sung, 1989) and Cho expanded the application region of EFFEA into coplanar coupled plates (Cho, 1993). Seo performed EFFEA of multi-dimensional built-up structures composed of a Euler beam, a Kirchhoff plate, and various structural joint conditions (Seo, 2005). Zhang et al. also performed EFFEA of various systems (Zhang et al., 2005a; 2005b).

Although EFA is a more advanced vibrational prediction tool than SEA, various efforts have been made to improve the theory of EFA. Langley showed that the standard two-dimensional governing differential equation of EFA based on plane wave components would be valid only for a structure having a highly reverberant wave field (Langley, 1991; Langley, 1995). Consequently, Kim et al. suggested a modified governing differential equation of EFA in a cylindrical coordinate system for predicting the energetics in a non-diffuse field (Kim et al., 1994). Additionally, many studies based on energetics have been performed (Smith, 1995; Le Bot, 1998a; 1998b; Lase et al., 1990; Lase et al., 1996).

Until now, although the plate in various general built-up structures is the most typical structural component, EFA for coupled Mindlin plate structures has not been performed. Especially, the significance of the Mindlin Plate Theory (MPT) in offshore oil-and-gas and ship building industries is emphasized. In this paper, EFA for coplanar coupled Mindlin plates was performed. The general power transfer relationship among all propagating out-of-plane waves existing in arbitrary coplanar coupled Mindlin plates was newly derived. For numerical validations, the classical solution for coplanar coupled Mindlin plates was obtained and was successfully compared with an EFA solution over various conditions.

ENERGY FLOW MODEL FOR OUT-OF-PLANE MOTION OF MINDLIN PLATE

The governing equation of out-of-plane motion in the loaded Mindlin plate with hysteresis damping is expressed by Fahy (1990)