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Approximate natural vibration analysis of rectangular plates with openings using assumed mode method

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ABSTRACT: *Natural vibration analysis of plates with openings of different shape represents an important issue in naval architecture and ocean engineering applications. In this paper, a procedure for vibration analysis of plates with openings and arbitrary edge constraints is presented. It is based on the assumed mode method, where natural frequencies and modes are determined by solving an eigenvalue problem of a multi-degree-of-freedom system matrix equation derived by using Lagrange's equations of motion. The presented solution represents an extension of a procedure for natural vibration analysis of rectangular plates without openings, which has been recently presented in the literature. The effect of an opening is taken into account in an intuitive way, i.e. by subtracting its energy from the total plate energy without opening. Illustrative numerical examples include dynamic analysis of rectangular plates with rectangular, elliptic, circular as well as oval openings with various plate thicknesses and different combinations of boundary conditions. The results are compared with those obtained by the finite element method (FEM) as well as those available in the relevant literature, and very good agreement is achieved.*

KEY WORDS: Natural vibration analysis; Flexural vibrations; Assumed mode method; Lagrange's equations; Rectangular plate; Plate opening; Arbitrary boundary constraints.

INTRODUCTION

Dynamic analysis of plates with openings represents an important issue in naval architecture and civil, mechanical as well as ocean engineering. Problem of determining flexural natural frequencies of plates with central free openings has been studied by many researchers (Leissa, 1969; Monahan et al., 1970; Bathe et al., 1973; Paramasivam, 1973; Hegarty and Ariman, 1975; Ali and Atwal, 1980; Laura et al., 1987; Gutierrez et al., 1987). Nowadays, the finite element method gives complete solution to a problem of vibration of plates with openings, and although it applied for very long time (Monahan et al., 1970), it still suffers from the computational time consumption. Finite difference method has been applied to a plate with opening and simply supported and clamped edges by Paramasivam (1973). Reddy (1982) analysed large amplitude flexural vibrations of layered composite plates with openings. Grossi et al. (1997) applied optimized Rayleigh-Ritz method to generate values of the fundamental frequency coefficient corresponding to the first fully antisymmetric mode for rectangular plates with circular holes and elastically restrained against rotation. In all these references positions of the openings are limited to plate central part. However, finite difference method has been applied to a plate with more than two openings by Aksu and Ali (1976). Huang and Sakiyama (1999) proposed approximate method for free vibration analysis of rectangular plates with openings of different shapes (circular,

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semi-circular, elliptic, square, rectangular, triangular, etc.). They consider a plate with an opening as a body with non-uniform thickness, where the opening is treated as an extremely thin part of the plate. In that sense the problem is translated into the free vibration analysis of the equivalent rectangular plate with non-uniform thickness by applying the discrete solution (Huang and Sakiyama, 1998). It should be noted that the problem is solved for arbitrary position of the opening within the plate area. An extensive literature survey on vibration analysis of rectangular plates with openings, with particular emphasis to application of Rayleigh-Ritz method, has been presented by Kwak and Han (2007). They proposed a Rayleigh-Ritz based method with different coordinate systems for a plate and opening with particular aim to simplify integration process in total energy determination.

Recently, assumed mode method using characteristic orthogonal polynomials is applied to problem of free vibration of rectangular plates of arbitrary edge constraints (Chung et al., 1993; Kim et al., 2012). In this paper, assumed mode method is applied to natural vibration analysis of plates with openings of different shape and arbitrary boundary conditions. The opening effect is accounted for by subtracting its strain and kinetic energies from corresponding total plate energies without openings. Natural frequencies and corresponding mode shapes are obtained by solving an eigenvalue problem of a multi-degree-of-freedom system matrix equation, which has been derived by Lagrange's equations of motion. It should be noted also that the proposed method can be applied for arbitrary number of openings and their arbitrary positions. Numerical examples include analysis of natural vibrations of rectangular plates with rectangular, elliptic, circular and oval openings. Both, thin and thick plates are considered. The results are compared with those obtained by the finite element method as well as some results available in the relevant literature. It is shown that the presented procedure can be used as a reliable alternative to widely used FEM, which may require much more time in model preparation, as for instance in case of plate with oval opening.

MATHEMATICAL MODEL

Equations of motion

The Mindlin thick plate theory, which takes shear influence and rotary inertia into account, is introduced (Mindlin et al., 1956). In that way the method can be applied not only to thin plates, but also to moderately thick and thick ones. The Mindlin theory deals with three general displacements, i.e. plate deflection w, and angles of cross-section rotation about x and y axis, ψ_x and ψ_y, respectively. From the equilibrium of sectional forces (bending moments, torsional moments and shear forces) and inertia forces, equations of motions are derived:

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}-D\left(\frac{\partial^2\psi_x}{\partial x^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_x}{\partial y^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_y}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial x}-\psi_x\right)=0 \tag{1}$$

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_y}{\partial t^2}-D\left(\frac{\partial^2\psi_y}{\partial y^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_y}{\partial x^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_x}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial y}-\psi_y\right)=0 \tag{2}$$

$$\frac{\rho}{kG}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial\psi_x}{\partial x}+\frac{\partial\psi_y}{\partial y}=0 \tag{3}$$

where ρ is plate density, h is plate thickness, k is shear coefficient, while ν is Poisson's ratio. Further on, D represents plate flexural rigidity $D=Eh^3/\left(12\left(1-\nu^2\right)\right)$, while E and $G=E/\left(2\left(1+\nu\right)\right)$ are Young's and shear modulus, respectively.

The basic idea to account for an opening effect is to subtract the energy of the opening part from the total plate energy without openings. In that sense, following expressions for the strain and kinetic energy of rectangular plate with openings, respectively, are valid: