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Computation of the inviscid drift force caused by nonlinear waves on a submerged circular cylinder

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ABSTRACT: *In this paper, we focused on computing the higher-harmonic components of the transmitted wave passing over a submerged circular cylinder to show that it is causing a horizontal negative drift force. As numerical models, a circular cylinder held fixed under free surface in deep water is adopted. As the submergence of a circular cylinder decreases and the incident wavelength becomes longer, the higher-harmonic components of the transmitted wave starts to increase. An increase of the higher-harmonic components of the transmitted wave makes the horizontal drift force be negative. It is also found that the higher-harmonic amplitudes averaged over the transmitted wave region become larger with the increase of wave steepness and wavelength as well as the decrease of submergence depth.*

KEY WORDS: Higher-harmonic component; Numerical wave tank; Drift force; Fully nonlinear potential flow; Transmitted wave.

INTRODUCTION

According to potential flow theory and the so-called d'Alembert paradox, there is no force acting on a submerged body in a steady state irrotational flow of an inviscid incompressible fluid. In such a case, only a viscous drag force can occur. For unsteady potential flows, such as caused by waves, however, a mean drift force can be induced on a submerged body. In particular, it has been long known that a negative drift force may be caused on submerged bodies by surface gravity waves of sufficient steepness, i.e., nonlinearity; hence, this wave-induced drift force is due to higher-order effects. Using conformal mapping, Dean (1948) thus found that a submerged circular cylinder does not reflect waves to leading order of steepness. Ursell (1950) confirmed this result by deriving the complete linear solution using a multipole expansion method. Following Ursell's approach and estimating second-order effects from linear results, Ogilvie (1963) showed the existence of a mean second-order vertical force, but found that the horizontal mean force vanished to second-order. Using a Stokes expansion, Vada (1987) solved the second-order diffraction problem in the frequency-domain, but could not calculate all the terms of the mean horizontal force. Longuet-Higgins (1977) observed in experiment that a freely moving, neutrally buoyant, submerged cylinder experienced a negative drift force, causing it to move towards the wavemaker. He attributed this force mostly to wave breaking and, to a lesser degree, to the second-harmonic component of the transmitted wave. This conclusion, however, is not corroborated by Miyata et al. (1988) and Inoue and Kyozuka (1984) measurements, who both found that, as the cylinder was moved closer to the free surface, causing more intense wave breaking, the negative horizontal drift force was actually reduced and ultimately even changed sign.

A number of two-dimensional, fully-nonlinear, inviscid time-domain computations have been proposed, to estimate strongly nonlinear effects caused by waves passing over submerged bodies of small equivalent diameter but large dimension in the transverse direction, with respect to wavelength, such as pipelines. Using the mixed Eulerian-Lagrangian method, Cointe (1989) calculated higher-order harmonic forces and wave transmission coefficients on a submerged cylinder, in a fully-nonlinear potential flow model, but did not calculate the horizontal drift force. Torum and Gudmestad (1990) computed particle trajectories and Lagrangian transport caused by steep waves, represented by exact streamfunction Stokes waves, over a submerged cylinder, in a space-periodic version of Grilli et al. (1989) fully-nonlinear potential flow model. Liu et al. (1992) applied the Higher-Order Spectral Method (HOS) to this problem and compared computations with analytical results and experimental observations. They also used exact deep-water Stokes waves as initial conditions and specified periodic conditions for upstream and downstream boundaries,

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a requirement of the HOS method.

In this paper, we establish the origin of the negative drift force caused by steep waves on a submerged cylinder, by similarly performing two-dimensional (2D) Fully Nonlinear Potential Flow (FNPF) simulations in the time domain. In the simulations, we use the most recent version of the model originally developed by Grilli et al. (1989), with improvements and additions by Grilli and Subramanya (1996) and Grilli and Horrillo (1997) (hereafter referred to as 2D-FNPF model). Unlike earlier studies, our computations are not space-periodic but feature the generation of exact fully nonlinear periodic incident waves at one extremity of a "Numerical Wave Tank" (NWT), as well as wave absorption/radiation at the other extremity. Although our model can simulate overturning waves, we did not consider wave breaking effects in this paper. For non-breaking waves, we will show that the higher-harmonic components of the transmitted wave are the main cause for the negative horizontal drift force on a submerged body. Numerical results will also show that the magnitude of this higher-harmonic components increases as the body submergence decreases, and incident wavelength and steepness increase.

NEGATIVE DRIFT FORCE

To establish the relationship between horizontal drift force and higher-harmonic components of the transmitted waves passing over a submerged body, it is useful to first obtain a simple estimate of the solution based on the conservation of energy and linear horizontal momentum. Assuming wave reflection by a submerged circular to be negligible and considering the incident and transmitted wave amplitudes on the downstream up/down sides of a submerged cylinder, let a_n, b_n be the n-th harmonics of the incident and transmitted wave amplitudes, respectively. With this assumption, application of the conservation of horizontal momentum gives an expression for the horizontal drift force to leading order, as,

$$D_x = \frac{\rho g}{4}\sum_{n=1}\left(a_n^2 - b_n^2\right) \tag{1}$$

From conservation of energy, a_n and b_n are related by,

$$\sum_{n=1}\frac{a_n^2}{n} = \sum_{n=1}\frac{b_n^2}{n} \tag{2}$$

Since, for periodic incident waves, the amplitude of the first harmonic a_1 is much greater than all other harmonic amplitudes, we can neglect all a_n , $n > 1$ term in Eqs. 1 and 2. From Eqs. 1 and 2, we can obtain

Eq. 3 provides a way to estimate D_x for given transmitted wave harmonic amplitudes, and is a generalization of the result of Longuet-Higgins (1977), who only considered the first (b_2) term only. Although it only represents an approximation valid for small incident wave steepness, Eq. 3 indicates that the horizontal force is always negative, with a magnitude that increases with the degree of higher-harmonic generation.

$$D_x = -\frac{\rho g}{4}\sum_{n=2} b_n^2 \frac{n-1}{n} \tag{3}$$

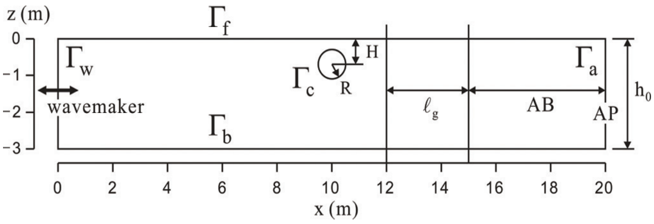

Fig. 1 Computational model for the nonlinear wave diffraction by a fixed submerged cylinder (AB: absorbing beach, AP: absorbing piston).

OVERVIEW OF NUMERICAL MODEL

Governing equations and numerical algorithms

Equations for the 2D-FNPF wave model are briefly presented in the following. The velocity potential ϕ (x, t) is used to describe inviscid irrotational flows in the vertical plane (x, z) and the velocity is defined by, $u = \nabla\phi = (u, w)$. Continuity equation in the fluid domain $\Omega(t)$ with boundary $\Gamma(t)$ is a Laplace's equation for the potential (Fig. 1),

$$\nabla^2\phi = 0 \qquad \text{in } \Omega(t) \tag{4}$$

On the free surface $\Gamma_f(t)$, ϕ satisfies the kinematic and dynamic boundary conditions,

$$\frac{D\boldsymbol{r}}{Dt} = \left(\frac{\partial}{\partial t} + \boldsymbol{u}\nabla\right)\boldsymbol{r} = \boldsymbol{u} = \nabla\phi \qquad \text{on } \Gamma_f(t) \tag{5}$$

$$\frac{D\phi}{Dt} = -gz + \frac{1}{2}\nabla\phi\cdot\nabla\phi - \frac{p_a}{\rho} \qquad \text{on } \Gamma_f(t) \tag{6}$$

respectively, with $\boldsymbol{r}$, the position vector on the free surface, g the gravitational acceleration, z the vertical coordinate, P_a the pressure at the free surface, and ρ the fluid density. Along the stationary bottom Γ_b and cylinder boundary Γ_c, the no-flow condition is prescribed as,

$$\frac{\partial\phi}{\partial n} = \nabla\phi\cdot\boldsymbol{n} = 0 \qquad \text{on } \Gamma_b \text{ and } \Gamma_c \tag{7}$$

where $\boldsymbol{n}=(n_x, n_z)$ is the outwards normal vector defined on the boundary. Boundary conditions for wave generation on boundary Γ_w and wave absorption on boundary Γ_a are presented in the next sections.