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Stochastic ship roll motion via path integral method

G. Cottone[1], M. Di Paola[1], R. Ibrahim[2], A. Pirrotta[1] and R. Santoro[1]

[1]*Dipartimento di Ingegneria Strutturale Aerospazialee Geotecnica,Viale delle Scienze, Palermo, Italy*
[2]*Wayne State University, Department of Mechanical Engineering, Detroit, USA*

ABSTRACT: *The response of ship roll oscillation under random ice impulsive loads modeled by Poisson arrival process is very important in studying the safety of ships navigation in cold regions. Under both external and parametric random excitations the evolution of the probability density function of roll motion is evaluated using the path integral (PI) approach. The PI method relies on the Chapman-Kolmogorov equation, which governs the response transition probability density functions at two close intervals of time. Once the response probability density function at an early close time is specified, its value at later close time can be evaluated. The PI method is first demonstrated via simple dynamical models and then applied for ship roll dynamics under random impulsive white noise excitation.*

KEY WORDS: Ship roll; Random impulsive ice loading; Poisson distribution; Path integral; Parametric random excitation; Chapman-Kolmogorov equation.

INTRODUCTION

The influence of floating ice on the dynamic behavior of ships and offshore structures depends on many factors such as ice thickness and its relative speed with respect to the floating structure. The ice resistance to ship motion forms an essential problem in ship design and navigation. Furthermore, local or global ice loads acting on ocean systems are random and non-smooth when impact interaction takes place. Impact loads on the bow of a ship navigating in solid ice may be modeled by a Poisson law. The assessment of ice related problems encountered by offshore structures as well as by ships during their navigation has recently been documented by Ibrahim et al. (2007). In view of ice loads on marine systems, new design regulations have been introduced by international organizations that are involved in the design and building of ships as well as offshore structures.

Ice loads acting on ocean systems are random in nature and have non-smooth characteristics when they are of impact type. In full-scale experiments, measurements of ice local and global loads revealed randomness in the ice forces and pressures (see, e.g., Meyerhoff and Schlachter, 1980; and Timco and Johnston, 2004). In some cases, ice loads are of impact type and have been assumed as a Poisson arrival process of loading events. Thus, one must deal with probabilistic approaches when studying ships' stochastic stability, response, and reliability. The treatment of dynamical systems under Poisson random processes has been considered in references (e.g., Köylüoğlu, et al., 1995; Di Paola and Pirrotta, 1999; and Proppe, 2003).

For systems under normal or non-normal white noise, the response statistics may be obtained by solving the Fokker-Plank Kolmogorov (FPK) equation or the Kolmogorov-Feller equation. However, exact solutions of the partial differential equations governing the evolution of the response probability density function (*pdf*) are known only for very few cases as shown by Caughey and Dienes (1961), Dimentberg (1982) and Vasta, 1995). Alternatively, several approximate solutions techniques have been developed including variational methods based on eigenfunction expansion of the transition *pdf* (Atkinson, 1973), finite element method (Spencer and Bergman, 1993) and the path integration approach (Köylüoğlu et al., 1995). The PI approach is an effective tool for evaluating the response in terms of probability density at each time instant, for evaluating moments of various orders, energy response *pdf*, first passage time of strong nonlinear systems. This approach is based on rewriting the FPK equation in integral form in which the kernel is the transition probability density function. Thus one can evaluate the response *pdf* at time ($t+\tau$) when its value at an early close time instant (t) is already specified. The crucial point is to define the kernel according to the system under investigation. In the case of normal white noise, if τ is small, even if the system is nonlinear, the transition *pdf* is almost Gaussian (short-time Gaussian approximation). It follows that the kernel of the integral form is Gaussian and this simplifies the analysis as shown by Barone et al. (2008). The accuracy of this method was validated using Monte Carlo simulation and the exact solution when the latter is available. Recently, Di Paola and Santoro (2008) have studied the case of Poisson white noise by evaluating the conditional

Corresponding author: *Raouf A. Ibrahim*
e-mail: *ibrahim@eng.wayne.edu*

probability density function (*cpdf*) in order to apply the PI method also for these systems.

The path integral technique was applied to the roll nonlinear motion of a ship in irregular waves by Kwon et al. (1993). The exciting moment due to the irregular waves was modeled as a non-white noise. Both damping and nonlinear restoring functions were included with the equivalent white-noise intensity. Lin and Yim (1995) developed a stochastic analysis scheme to examine the properties of chaotic roll motion and capsize of ships subjected to periodic excitation with a random noise disturbance. The associated Fokker-Planck equation governing the evolution of the probability density function of the roll motion was numerically solved by the path integral solution procedure to obtain joint probability density functions in state space. It was found that the presence of noise enlarges the boundary of the chaotic domains and bridges coexisting attracting basins in the local regimes. The probability of capsize was considered as an extreme excursion problem with the time-averaged probability density function as an invariant measure. Another version of the path integration approach based on the Gauss-Legendre quadrature integration rule was proposed by Gu (2006). It was applied for estimating the probability density of the nonlinear roll motion of ships in stochastic beam seas. The ship roll motion was described by a nonlinear random differential equation that includes a nonlinear damping moment and restoring moment. The results include the time-evolution of the ship response probability density as well as the tail region, where the probability value is very important for the system reliability analysis.

The case of small ships with water on deck subjected to random beam waves described by to a periodic force and white noise perturbation was considered by Liqin and Yougang (2007) using the path integral solution. This type of ship motion is governed by two dynamical regions: homoclinic and heteroclinic, where the heteroclinic model emulates symmetric vessel capsize and the homoclinic model represents a vessel with an initial bias caused by water on deck. The random Melnikov mean square criterion was used to determine the parameter domain for the ship's stochastic chaotic motion. The evolution of the probability density function of the roll response was calculated by solving the stochastic differential equations using the path integral method. It was found that in the probability density function of the system has two peaks for which the response of the system was found to jump from one peak to another for large amplitudes of periodic excitation. Manotov and Naess (2009) developed a combined analytical-numerical approach referred to as the successive-transition method, which is essentially a version of the path-integration solution and is based on an analytical approximation for the transition probability density. The method was applied to one-dimensional nonlinear Ito's equation describing the velocity of a ship maneuvering along a straight line under the action of the stochastic drag due to wind or sea waves. It was also used for the problem of ship roll motion up to its possible capsizing. It was indicated that the advantage of the proposed successive-transition is that it provides an account for the damping matrix in the approximation.

The case of Gaussian white noise acting simultaneously with Poisson white noise has not yet been considered in the literature and the present work is an attempt to extend the PI method for this case. In particular, the method will be utilized to examine the ship roll oscillation under parametric normal white noise acting simultaneously with additive Poisson white noise.

PATH INTEGRAL METHOD

This section provides the general features of the PI method by adopting a simple nonlinear system driven by a white noise described by the one-dimensional equation:

$$\begin{cases} \dot{X}(t) = -\alpha X(t) + f(X,t) + W(t) \\ X(0) = X_0 \end{cases} \tag{1}$$

where $f(X,t)$ is a deterministic nonlinear function of the response $X(t)$ and time t, $\alpha$is a positive parameter and $W(t)$ is a white noise and X_0 is the initial condition that may be either deterministic or random (Gaussian or non-Gaussian).

The starting point of the PI method is the Chapman-Kolmogorov equation that holds true, because of the Markovian property of the response:

$$p_X(x,t+\tau) = \int_D p_X(x,t+\tau | \bar{x},t) p_X(\bar{x},t)\, d\bar{x} \tag{2}$$

The numerical implementation of the PI method requires selecting a computational domain D. It is convenient to select a symmetrical computational domain with a given maximum size, $x_{max} = |x_1|$, i.e., $-x_1 \leq x \leq x_1$. The size of the domain is identified by, first, running a Monte Carlo simulation with a low number of samples. Then, dividing the domain in a discrete number of intervals, n_x, for each grid point, the path integral from equation (2) can be evaluated. One has to evaluate the kernel in equation (2), which requires the conditional joint *pdf*. From the entire set of trajectories of the response process, $X(t)$, one has to select those deterministic values at time t, i.e., $\bar{x}$, hereafter denoted as $\bar{X}(\rho)$(see Fig.1), by solving of the following differential equation:

$$\begin{cases} \dot{\bar{X}}(\rho) = -\alpha \bar{X}(\rho) + f(\bar{X},\rho) + W(t+\rho) \\ \bar{X}(0) = \bar{x} \end{cases} \tag{3}$$

where $\bar{x}$ is a deterministic initial condition and $0 \leq \rho \leq \tau$. Note that the *cpdf* of equation (1) coincides with the unconditional *pdf* of equation (3) evaluated at τ, i.e.,

$$p_X(x,t+\tau | \bar{x},t) = p_{\bar{X}}(x,\tau) \tag{4}$$

Fig.1 demonstrates the significance of the *cpdf* of the stochastic process $\bar{X}(\rho)$ evaluated in $\rho=\tau$. These are the general features of the PI method. To this end the problem is